Semi-passive vibration control using shunted piezoelectric materials

By

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*ScSo : Histoire, Géographie, Aménagement, Urbanisme, Archéologie, Science politique, Sociologie, Anthropologie*
To my son Avesta

To my parents
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<tr>
<td>$A$</td>
<td>surface area</td>
</tr>
<tr>
<td>$A_u$</td>
<td>displacement damping</td>
</tr>
<tr>
<td>$c$</td>
<td>damping per unit length</td>
</tr>
<tr>
<td>$c_i$</td>
<td>generalised damping coefficient for mode $i$</td>
</tr>
<tr>
<td>$C$</td>
<td>capacitance</td>
</tr>
<tr>
<td>$C_p$</td>
<td>piezo- capacitance</td>
</tr>
<tr>
<td>$D$</td>
<td>electrical displacement or density of surface charge</td>
</tr>
<tr>
<td>$e$</td>
<td>piezoelectric constant</td>
</tr>
<tr>
<td>$E$</td>
<td>Young’s modulus of elasticity</td>
</tr>
<tr>
<td>$\bar{E}$</td>
<td>Electrical field</td>
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<td>$E^\bar{E}$</td>
<td>Young’s modulus under constant electrical field</td>
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<tr>
<td>$E_d$</td>
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<td>system energy</td>
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<tr>
<td>$E_t$</td>
<td>transferred energy</td>
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<tr>
<td>$f(t)$</td>
<td>excitation force</td>
</tr>
<tr>
<td>$f(x,t)$</td>
<td>excitation force per unit length</td>
</tr>
<tr>
<td>$f_s$</td>
<td>frequency of oscillatory circuit</td>
</tr>
<tr>
<td>$F_e$</td>
<td>external force</td>
</tr>
<tr>
<td>$F_i$</td>
<td>internal force</td>
</tr>
<tr>
<td>$F_p$</td>
<td>piezo-element force</td>
</tr>
<tr>
<td>$F_{pd}(x)$</td>
<td>probability distribution function</td>
</tr>
<tr>
<td>$h$</td>
<td>piecewise piezo-voltage function</td>
</tr>
<tr>
<td>$I_{out}$</td>
<td>outgoing current from the piezo-element</td>
</tr>
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</table>
$I_u$ mean squared response of displacement
$J$ inertia moment of the cross-section
$k$ global electromechanical coupling coefficient
$k_i$ generalised electromechanical coupling coefficient for mode $i$
$k_p$ electromechanical coupling coefficient
$K_{de}$ open-circuit equivalent piezo-element stiffness
$K_{di}$ open-circuit generalised stiffness for mode $i$
$K_{ei}$ short-circuit generalised stiffness for mode $i$
$K_{pe}$ short-circuit equivalent piezo-element stiffness
$K_i$ modal stiffness for mode $i$
$l$ beam length
$l_e$ work done by external force
$l_i$ dissipated energy
$L$ inductance
$L_p$ piezo-element thickness
$M$ mass
$M_i$ modal mass for mode $i$
$P$ polarization
$P_{sw}$ probability
$q_i$ modal coordinate
$Q$ outgoing electrical charge from the piezo-element
$Q_i$ electrical quality factor
$\bar{Q}_i$ generalised excitation force
$Q_{mi}$ mechanical quality factor for mode $i$
$R$ resistance load
$R_i$ inductance resistance
$S$ strain
$T$ period of the fundamental resonance frequency
$T_{es}$ sliding time window
$T_s$ stress
$u$ structural displacement (deflection)
$u_m$ displacement threshold
\[ u_M \] displacement amplitude
\[ (u_M)_c \] controlled amplitude of deflection
\[ (u_M)_{unc} \] uncontrolled amplitude of deflection
\[ u_p \] piezo-element deformation
\[ v \] piezoelectric voltage
\[ v_{es} \] estimated piezo-voltage
\[ v_m \] voltage threshold
\[ v_{pt} \] potential energy per unit mass
\[ v_s \] displacement sensor voltage
\[ V_c \] continuous voltage source for SSDV technique
\[ V_{ext} \] external applied voltage
\[ w \] periodic absorbed energy
\[ W_e \] electrical energy
\[ W_m \] total elastic energy
\[ W_{mp} \] piezoelectric elastic energy
\[ Z \] shunt circuit impedance
\[ \alpha \] force factor
\[ \alpha_i \] macroscopic piezoelectric coefficient for mode \( i \)
\[ \beta \] tuning coefficient
\[ \beta_v \] proportionality coefficient between open-circuit piezo-voltage and voltage source \( V_c \) for SSDV technique
\[ \gamma \] Inversion coefficient
\[ \Delta t \] dSPACE step time
\[ \Delta t_s \] inversion time
\[ \varepsilon \] permittivity of piezo-element under constant strain
\[ \zeta_i \] damping ratio for mode \( i \)
\[ \theta \] phase angle
\[ \lambda_i \] electromechanical proportionality coefficient
\[ \mu_x \] temporal average of \( x \)
\[ \rho \] Mass per unit length
\[ \sigma_x \] standard deviation of \( x \)
\[ \nu \] Poisson ratio
\( \varphi_i \) eigenmode function
\( \psi \) specific damping coefficient
\( \psi_x \) root mean square of \( x \) (rms)
\( \omega \) excitation force frequency
\( \omega_{li} \) open-circuit angular frequency for mode \( i \)
\( \omega_{ei} \) short-circuit angular frequency for mode \( i \)
\( \omega_x \) angular frequency of oscillatory circuit
Résumé de thèse

Tous les systèmes mécaniques sont soumis à diverses conditions dont peuvent résulter des vibrations. Ces vibrations conduisent souvent à la fatigue des matériaux, entraînant des dommages structurels, la détérioration des performances du système et l'augmentation du niveau de bruit transmis. De telles vibrations ne peuvent être tolérées et, par conséquent, l'élaboration d'une stratégie pour réduire ces vibrations a été un des grands axes de la recherche. L’amortissement des vibrations est une des manifestations de la perte d’énergie mécanique liée au mouvement du système. Les transducteurs piézo-électriques, en liaison avec des réseaux électriques appropriés peuvent être utilisés comme dispositifs de dissipation d'énergie mécanique. Les matériaux piézoélectriques sont ajoutés au système d'origine pour absorber les vibrations. Dans ce cas, la réduction des vibrations du système d’origine est obtenue en canalisant l’énergie aux matériaux piézoélectrique. Ces matériaux sont caractérisés par la capacité de transformation de l'énergie de déformation en énergie électrique et vice versa. En raison de leurs caractéristiques, ils sont au centre de l'attention de différentes recherches qui concernent la détection et le contrôle du mouvement des structures. Ils sont intégrés à l’intérieur des matériaux ou colles sur les structures et connectés au dispositif de surveillance pour détecter les mouvements, ou au shunt circuit pour l'amortissement des vibrations ou la récupération d'énergie dans ces structures. L’amortissement électronique qui utilise la céramique piézoélectrique est moins sensible que d’autres à la température et plus adaptatives. Dans cette technique d’amortissement, l'énergie mécanique de la structure est convertie en énergie électrique en utilisant des matériaux piézoélectriques. L'énergie électrique à son tour, est dissipée sous forme de chaleur, dans un shunt circuit électrique. En général, il existe deux grandes catégories de techniques de contrôle des vibrations et de l’acoustique qui utilisent les éléments piézoélectriques: les techniques passives et les techniques actives. D’excellents résultats sont obtenus par la technique active de contrôle, mais le système est un grand consommateur d'énergie et nécessite du matériel lourd et coûteux. C’est un système complexe composé d’un capteur de mesure des vibrations, d’un calculateur pour déduire la tension à appliquer sur les éléments piézoélectriques actifs, et en fin des générateurs et amplificateurs de tension pour exciter ces actionneurs. La nécessité de
recourir aux techniques d'amortissement passif résulte de la complexité qui est apportée aux structures par les techniques de contrôle actif. Le travail présenté dans ce manuscrit permet de réaliser des matériaux intelligents (matériaux piézoélectriques) pour le contrôle des vibrations. Les travaux de cette thèse concernent l'étude d'une technique particulière se rapportant au traitement de la tension générée par les éléments piézoélectriques. Cette technique appelée Synchronised Switch Damping (SSD) a été développée par le LGEF (laboratoire de génie électrique et ferroélectricité de l'INSA de Lyon) consiste en un traitement non linéaire de la tension de matériaux piézoélectrique, ce qui induit une augmentation de la conversion d'énergie électromécanique. Ces techniques semi-passives sont une alternative intéressante aux systèmes actifs en particulier lorsque les critères de volume et le coût sont importants. Ces dispositifs ont connu une forte évolution au cours de ces dernières années, en raison de leur performances et avantages par rapport aux techniques actives et passives. Plus précisément, SSD (Synchronised Switch Damping) et ses techniques dérivées, qui ont été développés dans le domaine de la piézoélectricité d'amortissement, ont conduit à un très bon compromis entre la simplicité, l'énergie nécessaire et la performance. La technique de contrôle de la semi passive non linéaire (SSDI) est intéressante pour les applications de l'amortissement des vibrations, car elle présente simultanément de bonnes performances de l'amortissement, en particulier dans les basses fréquences d'excitation contrairement aux matériaux viscoélastiques, une bonne robustesse et ne consomme qu'une très faible puissance. Il est moins sensible aux variations des paramètres du système et la température ambiante. Néanmoins, il est nécessaire d'avoir d'une petite quantité de l'énergie électrique externe, qui est utilisée uniquement pour le contrôle électronique afin d'activer les transistors. Toutefois le système peut être autoalimenté en utilisant la conversion l'énergie électrique des éléments piézoélectriques. Cela signifie qu'aucun câblage (système sans fil) ou d'alimentation en énergie sont nécessaires. L'avantage de cette méthode est la simplicité de son intégration que ne peut garantir une approche active. Si l'on compare avec la technique passive, l'amortissement est plus important. La simplicité et l'efficacité de technique semi passive permettent de multiples application dans divers domaines: contrôle acoustique, contrôle des vibrations ou la récupération d'énergie. Il a été montré que la loi sur le contrôle consistant à déclencher l'inversion de switch sur chaque extremum de la
tension (ou le déplacement) est optimale pour l'excitation harmonique. Toutefois, dans le cas complexe de vibrations induites soit par un choc ou par des chocs répétitifs aléatoires ou une excitation aléatoire appliquée à une structure, la loi de contrôle systématique précédente d écrite n'est pas très performante. Le problème rencontré est que de nombreux maxima locaux se produisent et il devient difficile de déterminer celui qui devrait être choisi pour déclencher la séquence de commutation afin de maximiser la tension piézoélectrique et de renforcer l'énergie d'extraction et de la performance de l'amortissement sur une large bande de la fréquence de vibrations. Une nouvelle loi sur le contrôle pour les dispositifs semi passifs d'amortissement de piézoélectrique est proposée dans ce travail. Dans ce cas, le signal de la tension ou le déplacement est analysé au cours d'une fenêtre de temps et le niveau du seuil de déplacement ou de tension statistiquement probable est déterminé à la fois par la moyenne et le standard déviation du signal au cours de la période d'observation. Une diminution significative de l'énergie des vibrations est démontrée expérimentalement et théoriquement dans le cas d'une poutre encastrée libre excitée par le bruit aléatoire. Ces résultats montrent que cette nouvelle stratégie de contrôle est très efficace. Il est montré que, dans le cas multimodal de vibrations, tous les modes sont contrôlés et de préférence ceux qui correspondent à la plus grande amplitude de déplacement. Cette technique non linéaire d'amortissement consiste à ajouter un dispositif de commutation en parallèle avec les éléments piézoélectriques. Le courant dans le dispositif de commutation est toujours égal à zéro, sauf au cours de l'inversion de tension qui a lieu à chaque gâchette de switch (un simple circuit composé d'un interrupteur électronique et une inductance) que l'élément piézoélectrique est brièvement branché à un shunt circuit de électrique qui peut être un simple court-circuit (SSDS), ou une petite inductance (SSDI) à des moments précis dans le cycle de vibrations de la structure. Dans ces cas, le traitement non linéaire de la tension crée une force mécanique sous forme de fonction morceaux qui est en opposition de phase avec la vitesse de vibration. Cela signifie que du point de vue de structure le traitement non linéaire introduit un mécanisme de tel que le frottement sec. Par ailleurs, dans ce cas, il n'est pas nécessaire d'aucune d'informations sur les modes de la structure et il est très adaptable.

Afin d'analyser théoriquement les différentes méthodes des statistiques, une poutre encastrée libre, équipée avec des éléments piézoélectriques filaires sur une cellule de commutation SSDI a été simulée et expérimenté. Pour les études expérimentales, la
poutre est excitée au moyen d'un électroaimant. Un élément piézoélectrique est utilisé comme un capteur alors que d'autres patches sont utilisés comme actionneurs. La technique de contrôle est mise en œuvre en utilisant dSPACE digital signal processing et un logiciel de modélisation (Simulink).

Dans le cas de grandes bandes d'excitation, l'optimisation des éléments piézoélectriques (taille et emplacement) ainsi que le switching réseau n'est pas suffisant. Dans ce cas, de nombreux extrema apparaissent sur la tension et la déformation, ce qui correspond aux différents modes de la structure. Ensuite, afin d'optimiser l'amortissement des vibrations, nous avons concentré nos travaux sur l'étude et le développement de technique semi passives basées sur l'analyse de probabilité et de statistique, qui peuvent permettre de définir un critère pour identifier plus précisément les instants de commutation. Ces nouvelles techniques modifient brièvement les conditions électriques, augmentant de cette manière l'énergie électrostatique sur l'électrode de la piézoélectrique à être toujours en hausse, sauf pendant la phase de commutation. Les développements récents ont été axés sur la stratégie optimale du commutateur pour amortir les vibrations ou pour la récupération d'énergie. Les stratégies de contrôle de probabilité et des statistiques sont fondées sur l'idée que celles-ci permettent à la tension piézoélectrique d'atteindre une valeur significative (\(v_m\)) qui est statistiquement probable avant de permettre soit d'inverser la tension (SSDI) ou de la forcer à zéro (SSDS). En plus, la gâchette de switch sera effectuée sur le maximum local, pour lequel l'énergie stockée sur l'élément piézoélectrique est maximum. Les outils numériques et expérimentaux sont conçus pour vérifier sur cette idée. Il semble que la stratégie proposée marche bien pour divers types de forces d'excitation. Il a également été montré que pour de plus fins amortissement de meilleurs résultats pourrait être obtenus en utilisant soit une image du déplacement ou le carré de cette valeur.

La capacité de réduire l'amplitude des vibrations sur une large bande de fréquence est essentielle dans le cas de contrôle des vibrations. Ici, elle est suivie par une discussion sur la stratégie de contrôle semi passive des vibrations en fenêtre glissant de temps et la méthode statistique proposée. Cette étude montre que cette nouvelle stratégie développée permet une facilité de mise en œuvre de cette technique d'amortissement pour tous les types des excitations. Les résultats obtenus sur une poutre
encastrée libre lorsqu'elle est soumise à des impulsions, excitation aléatoire stationnaire et nonstationnaire ont été pris en considération. Dans ce cas, le switching séquence est mise en œuvre basée sur l'analyse statistique du signal de la déformation sur une fenêtre glissant de temps. Dans chaque instant, juste avant l'instant présent, la moyenne μ et la déviation standard temporelle σ sur une fenêtre glissant de temps \( T_{es} \) sont calculées et le probable seuil de déformation est statistiquement déterminé. En fait, ce calcul en chaque instant du temps sur la fenêtre glissant est pour prendre une décision au prochain switch. Cette approche dans le cas de vibration aléatoire est très efficace. Dans ce cas, la caractéristique la plus évidente est qu'il s'agit de nonperiodic. La connaissance de l'histoire passée du mouvement aléatoire est suffisante pour prédire la probabilité d'occurrence de diverses valeurs des déplacements, mais insuffisante pour prévoir de façon précise l'ampleur à un instant précis. La performance de la méthode statistique est très bien, car l'histoire du signal dans la fenêtre de temps passé \( T_{es} \) est étudié deux fois, par la moyenne et le standard déviation du signal. Donc la détection des extrénums dans ce cas est meilleure, en particulier dans le cas de vibrations aléatoires.

L'approche proposée pour la technique de contrôle non linéaire semi passive (SSDI) démontre une amélioration des performances sur la quasi-totalité de la gamme de fonctionnement prévu. Les résultats montrent que cette méthode de contrôle n'est pas très sensible à tous les types de comportements d'excitation (aléatoire stationnaire ou nonstationnaire et impulsion). Parce que, la fenêtre glissante est toujours en mouvement avec le signal et avec les variations du signal dans le temps, les valeurs statistiques changent ainsi proportionnellement, Par conséquent, les moments pour décider de déclencher l'interrupteur sont presque optimaux. On peut dire que la fenêtre glissante est indépendante du type de signal d'excitation. Cette caractéristique est très importante dans la pratique : Il a été démontré expérimentalement et théoriquement une diminution importante de l'énergie de vibration. Ensuite, l'analyse statistique de la déformation pourrait permettre la définition de critères permettant d'identifier plus précisément les instants de commutation. De plus, en raison du filtrage du signal d'excitation par le système, la détection extrénum est devenue plus facile en particulier pour les systèmes avec des valeurs plus élevées de \( Q_m \).

Aussi, l’amortissements par des élément piezoélectriques est une fonction de la taille de leurs surfaces et la valeur de la fréquence d'excitation. Lorsque la structure vibre avec de basses fréquences d'excitation, une plus grande surface d’éléments
piézoélectriques est plus efficace et quand elle vibre à haute fréquence d'excitation moins de surface est nécessaire, donc plus économique. En général, avec l'augmentation de la fréquence d'excitation l'amortissement des piézoélectriques diminue. La capacité d'amortissement de l'interrupteur (piézoélectriques) est doucement une fonction de la fréquence d'excitation. Par conséquent, la conversion d'énergie électromécanique diminue avec l'augmentation de la fréquence d'excitation. Les amortissements d'éléments piézoélectriques sont sensibles aux variations de l'amplitude d'excitation. Avec l'augmentation de l'amplitude d'excitation, la déformation des éléments piézoélectriques et piézoélectriques d'amortissement augmente, ainsi mais elle est limitée en ce qui concerne la taille de la surface des éléments piézoélectriques.

Aussi, dans ce manuscrit, l'effet des conditions de support sur l'amortissement des éléments piézoélectriques dans le cas de méthode SSDI est observé expérimentalement. Ensuite, la sensibilité du système liée à ces contraintes est étudiée. Il est constaté que la fréquence fondamentale est grandement dépendante de ces conditions. L'effet de ces contraintes est distribué dans tout le système et affecte de manière significative les résultats. La valeur d'amortissement des vibrations est proportionnelle à la valeur de la déformation de l'élément piézoélectrique. La déformation est le principal facteur de contrôle des vibrations par les matériaux piézoélectriques. Les conditions de support affectent l'énergie de déformation dans le système. De même, des conditions limites pourraient avoir des incidences importantes sur les résultats et la valeur de sensibilité à ces conditions serait surprenante. Ces effets sont l'équivalent dynamique du principe du Saint-Venant utilisé en mécanique des matériaux.

Afin d'améliorer la performance de cette technique non linéaire, la technique SSDV peut être utilisée. Mais, cette technique conduit à l'instabilité lorsque l'intensité de l'excitation mécanique diminue. Cet inconvénient est efficacement contourné par la technique SSDV modifiée. Cela peut être fait avec la méthode simple que devrait être proportionnées et en signe opposé au déplacement afin d'éliminer le risque d'instabilité. La technique SSDV accroît l'efficacité de la technique SSDI.

Cette technique non linéaire est également mise en place sur les systèmes de récupération d'énergie (synchronised switch harvesting (SSH)). La technique SSD peut être utilisée pour la récupération d'énergie par des matériaux pyroélectricités.
Pour avoir une meilleure amélioration de l'amortissement, les différentes techniques peuvent être utilisées ensemble dans des vibrations (contrôle hybride), par exemple pour les systèmes avec les techniques SSD et des matériaux viscoélastiques. Les matériaux viscoélastiques seront utilisés pour amortir les hautes fréquences, tandis que la technique SSD se concentre sur les basses fréquences d'amortissement pour lesquels les matériaux viscoélastiques ne sont pas efficaces. Cette association sera intéressante en raison de son efficacité dans les basses fréquences et l'élimination des harmoniques générés par le traitement non linéaire.

Ce manuscrit contient sept chapitres. Le premier chapitre a été alloué à décrire les détails des différentes techniques de contrôle des vibrations. Il présente aussi brièvement les travaux effectués sur ces sujets. La méthode passive, active, et enfin, semi passive de contrôle avec leurs avantages et leurs limites sont discutées, ainsi que la motivation qui incite à faire la mise au point de contrôle par des matériaux intelligents (matériaux piézoélectriques). Le deuxième chapitre décrit les techniques d'amortissement de vibrations SSD, et la modélisation ainsi que le modèle analytique de la structure. Le troisième chapitre propose une nouvelle loi de contrôle permettant d'optimiser les techniques non linéaires dans le cas d’une grande bande d'excitation : des résultats théoriques et expérimentaux sont présentés. Le quatrième chapitre présente les performances de la technique SSDI utilisant la nouvelle approche proposée (fenêtre glissante de temps) pour les nombreux types de forces d'excitation, ainsi que les résultats théoriques pour les différentes structures. Dans le chapitre cinq, le comportement d'amortissement des matériaux piézoélectriques sont analysés ainsi que l'effet de surface des éléments piézoélectriques sur le l'amortissement dans les hautes et basses valeurs de paramètres d'excitation (amplitude et fréquence). L'effet des conditions aux limites sur d'amortissement piézoélectrique dans le cas des contrôles de vibrations semi passifs est représenté expérimentalement dans le chapitre six. Enfin, les conclusions générales sont présentées dans le chapitre sept.
Introduction

All mechanical systems are subjected to various conditions that may result in vibration motion. These vibrations often lead to material fatigue, structural damage and failure, deterioration of system performance and increase of noise level. These effects are usually prominent around the natural frequencies of the system. Such vibrations cannot be tolerated and, therefore, developing a strategy for reducing these vibrations has been a major focus of research. This subject has considerable industrial consequences. Vibrations and acoustic control are used in many domains such as automobiles, aeronautics, aerospace, machine tools, etc. In vibration control the aim is the amplitude limitation of the vibrating structure in order to improve the reliability of their functions. Vibration control of a structure also allows increasing its resistance to fatigue then increasing the duration of its live. One of the recent strategies of vibration and acoustic control is the control by intelligent materials. In general, there are two large categories of vibration and acoustic control techniques using the piezoelectric elements: passive techniques that consist of connecting the passive electrical circuits (capacity, inductance, resistance) to the piezoelectric elements. The electrical converted energy is dissipated in joule effect and the active techniques that use a calculator associate with the source of external energy to oppose vibrations. The need for passive damping techniques arises from the complexities that are brought to the structures by active control techniques as well as by the increase of weight and energy supplies required to implement such control systems. The use of passive shunting in damping the vibration of structures was studied in different researches that concluded its effectiveness. The work of this thesis concerns the study of a particular technique related to the treatment of generated voltage by the piezoelectric elements. This nonlinear technique increases the effect of electromechanical conversion of
piezoelectric materials considerably. This technique called Synchronized Switching Damping (SSD) has been developed in LGEF (laboratory of electrical engineering and ferroelectricity of INSA-Lyon). The SSD techniques qualify the semi-passives techniques. The advantage of these techniques is that can be self-powered by using the converted electrical energy by piezoelectric elements. They are simple to implement and they do not have the inconveniences of the active techniques such as cost and accumulation and they represent a good performance. The presented work in the manuscript propose a new approach of control for the SSD techniques that allowing to increase of damping in the case of complex vibration such as random excitations. This new approach for the vibration damping is the statistical approach on the piezoelectric voltage or displacement of structure. Numerical as well as experimental results are presented for a cantilever beam. These results show the ability of the piezoelectric patches with passive shunting to damp out the structural vibration and also show that this new strategy of control is very efficient. Moreover, it shows the effectiveness of the size of piezoelectric patches on the vibration damping in large bandwidth of the excitation frequencies as well as the variations of excitation amplitude.

For the experimental studies, the beam is excited using an electromagnet. A piezo-element is used as a sensor, while other piezoelectric patches are used as actuators. The control technique is implemented using dSPACE digital signal processing board and a modelling software (SIMULINK). This manuscript contains seven chapters. The first chapter has been allocated to describing the details of the different techniques of vibration control. It also presents briefly the works done on these subjects. Passive, active, and finally semi-passive control and their advantages and limitations are discussed. The second chapter describes the SSD damping vibration technique, and its modelling as well as the analytical model of structure. The third chapter proposes a new law of control allowing optimising the nonlinear techniques in the case of large band excitation. Theoretical and experimental results are presented. The fourth chapter presents the performance of SSDI technique using the new proposed approach (sliding time window) for the many types of excitation forces as well as the theoretical results for the different structures. Chapter five presents experimentally the effect of the size of piezo-elements on the SSD technique and studies the sensibility of the piezo-element damping to the variations of the excitation force parameters. Chapter six studies the
effect of boundary conditions on the SSDI technique and finally, Chapter seven discuss the general conclusions.
Chapter: 1

Literatures review and general concepts of different techniques of vibration control by intelligent materials

This chapter introduces the advantage of using piezoelectric materials in the control systems. In addition, the principal techniques of vibration control and their advantages and disadvantages are discussed as well as the motivation that incites the development of control by the smart materials.

1.1 Introduction

There are two fundamentally different primary methods of vibration protection. These are vibration isolation and vibration absorption. In vibration isolation, the original system is divided into two parts, which are connected by means of additional mechanical devices, such as springs and dampers. These devices are referred to as vibration isolators. Common examples include rubber mounts for machinery and shock absorbers for automobiles. Isolators are positioned between the source of disturbance and the object to be protected or, equivalently, between the object and its supporting base. Isolators reduce the magnitude of force transmitted from the vibrating object to its foundation or, equivalently, reduce the transmitted motion from the base to the object. On the other hand, another system is attached to the original system in vibration absorption. Thus, another degree of freedom is added to the system. The parameters of the attached system are chosen so as to cause a decrease in the vibration level of the
original system. These attached systems are referred to as vibration absorbers. Vibration absorbers reduce vibrations of the original system by channelling energy to the absorber itself. Piezoelectric materials are the one type of absorbers that are characterized by having the capability of transforming strain energy into electrical energy and vice versa. Piezoelectric materials have the capability of generating electric charges on their surfaces when mechanical loads are applied on them, which is known as the forward piezoelectric action or the sensing action. They also exhibit what is known as the reverse action, that is, when the piezoelectric material is subjected to an electric field, they undergo mechanical deformations. This is also known as the actuator action. Due to those two characteristics of the piezoelectric materials, they have been the centre of attention of different researches that were concerned with the sensing and the control of motion of structures. Piezoelectric materials are embedded into or bonded on the structures and connected to monitoring device to detect motions or shunt circuit to vibration damping or harvesting energy in those structures. Those sensors are very useful especially in detecting damages in structures or indicating excessive vibration in different locations. On the other hand, the actuators are placed on structures to impose controlled deformations for vibration control of different structural elements. The published literatures on the control using piezoelectric materials are vast. Most damping falls in two categories: passive damping and active damping. Recently, a new subset of damping was created by combining passive and active damping, thereby producing a hybrid damping treatment. Using of viscoelastic materials is very in circulation in vibration control but they have some limitations such as: damping coefficient depend on temperature, its volume is not negligible and limiting performance in low frequencies. Also, The temperature variations that can greatly influence the mechanical response of viscoelastic materials and on the other hand the change of their dynamic stiffness and damping properties with the excitation frequency cause other methods and materials or hybrid control techniques to be considered. The implemented damping by the electro active devices offer the interesting alternative allows in must cases to bypass of these limitations. Because they present the remarkable characteristics in term of electromechanical coupling, generally, the piezoelectric materials are very integrated in this type of control devices. Moreover, it should be noted that these devices can be
classified according to different methods of utilisation, in general based on the energetic criterions.

### 1.1.1. Piezoelectricity

Piezoelectricity is the ability of crystals and certain ceramic materials to generate a voltage in response to applied mechanical stress. When a mechanical stress is applied to the piezoelectric material, electrical charges appeared, and the voltage was proportional to the stress. The piezoelectric effect was reversible in that piezoelectric crystals, when subjected to an externally applied voltage, can deform by a small amount. An applied mechanical stress will generate a voltage and an applied voltage will change the shape of the solid by a small amount (Fig. 1-1). Piezoelectric effect describes the relation between a mechanical stress and an electrical voltage in solids or can be described as the link between electrostatics and mechanics. The piezoelectric effect occurs only in non conductive materials.

![Piezoelectric effects](image)

**Figure 1-1:** Piezoelectric effects: (a) Piezo-element, (b) Direct effect, (c) Converse effect.

The advantages of piezoelectric elements are lot: They are capable to convert the electromechanical energy reversibly by piezoelectric effect (direct or inverse). Then they can be indifferently used such as a sensor or an actuator or even accumulate of these two functions. This reversibility allows obtaining the advance systems (smart systems) that the requirements of an intelligent structure is classified into four main
categories; actuators, sensors, control methodologies, and controller hardware. The integration of these materials is very easy. They are available in different forms: massive elements (plates, columns…) films, fibres, soft composite elements …

The efficiency of control techniques that integrated on piezoelectric materials is relying on the electromechanical coupling coefficient of active structure. This coupling coefficient guarantees effectively the good conversion of energy, effective control and some consideration types of piezoelectric applications, such as sensors and actuators. This coefficient depend on certain number of parameters, in particular the volume, the properties and especially the position of active elements on the structure. Effectively, the extracted vibration energy from the structure is optimal when the active elements are situated in the area of high deformation and high stresses. This point maybe was a restricting factor, because it is necessary to have a good knowledge of vibration state of controlled structure.

The stick of piezoelectric elements can also be a blocking point on some structures. It should be the mechanical stress completely transmitted to active elements in order to obtain the good coupling coefficient. On the other hand, the sticking on the complex forms maybe was a problem which prevent good conformation of element on the structure. But apparition of the flexible piezoelectric tends to minimise of this problem.

1.1.2. Electromechanical coupling coefficient

The coupling coefficient of structure can be expressed as a function of geometry and the properties of materials. This may be very usable to optimise the electromechanical response of structure. In the classic case, where it is studied the structure coupling as a function of the thickness of piezoelectric patches, it is possible to find an optimal thickness that maximise the coupling [1]. The optimum damping of the piezoelectric elements is obtained by optimisation of the electromechanical characteristics, which is the optimisation of its electrical response to mechanical excitation. Electromechanical coupling of vibrated structure is a function of the excited modes. In general, the coupling coefficient can be defined for each mode as a constant coefficient. The square of the electromechanical coupling coefficient of structure in open-circuit defines as:

\[ k^2 = \frac{\text{converted mechanical energy to electrical charge}}{\text{supplied mechanical energy}} \]
Or:

\[ k^2 = \frac{\text{converted electrical energy to mechanical deformation}}{\text{supplied electrical energy}} \quad (1.1) \]

Or, it defines as a ratio of electrical energy \( W_e \) to total elastic energy \( W_m \). If \( W_{mp} \) and \( k_p \) are the elastic energy and coupling coefficient of piezoelectric materials, the square of global coupling coefficient \( k \) of structure is then given by Eq. (1.2) [1]. It is appeared that the maximisation of the global coupling coefficient is related to maximisation of the ratio of elastic energy in the piezoelectric materials to the total elastic energy.

\[ k^2 = \frac{W_e}{W_m} = k_p^2 \frac{W_{mp}}{W_m} \quad (1.2) \]

where \( k_p^2 = \frac{W_e}{W_{mp}} \).

In addition to dimensions of the piezoelectric inserts, their positions on the structure are very important to maximise the coupling coefficient. The optimization of the electromechanical coupling is a complex problem.

Number of scientific works were been realised on the subject of modelling and optimisation of the piezoelectric structure [2, 3]. Although, the actual generalized electromechanical coupling coefficient is difficult to calculate. But the generalized electromechanical coupling coefficients for each mode are experimentally determined from the structure’s open-circuit, \( \omega_{di} \) and short-circuit \( \omega_{ei} \) natural frequencies as [4]:

\[ k_i^2 = \frac{\omega_{di}^2 - \omega_{ei}^2}{\omega_{di}^2} \quad (1.3) \]

### 1.2 Various techniques of control

#### 1.2.1. Passive control

Passive control contain of all the techniques either based on the using of absorbing viscoelastic materials [5] to increase the damping coefficient of structure (vibration control), or on the using of the material to absorb the mechanical energy or radiated
acoustic energy (control acoustic). The common characteristic of these techniques is that they do not rely on an external source of energy. The passive techniques based on the piezoelectric materials are discussed below.

1.2.1.1. Passive damping behaviour and passively shunted piezoelectric material

In passive devices the vibration energy is handled without the need of external energy and in general these methods increase the intrinsic damping of structure. Many materials, such as steel, aluminium and other metals, used in engineering have very little inherent damping. Therefore any structure that is built using these materials has vibrational problems when excited near the resonance frequencies. Adding of materials, such as viscoelastic materials, increase the damping and reduce the vibration amplitude at resonance frequencies [6]. Vibration of the structure causes the viscoelastic materials to strain and consequently this strain energy causes the viscoelastic materials to increase in temperature. The dissipation of heat from the structure removes vibrational energy from the system and is referred to as damping. Damping reduces the magnitude and the time duration of the vibration. In order to have acceptable levels of damping, large amounts of viscoelastic materials must be used. Therefore, using the piezoelectric materials connected to dissipation shunt circuits is suitable. They have been used to add damping to vibrating structures for over two decades [7]. As the piezoelectric material can be considered as a transformer of energy, from mechanical to electrical and vice versa, a part of the electrical energy generated by that transformer could be allowed to flow in a circuit that is connected to the piezoelectric patch. The dissipation characteristics of this circuit would be determined by its electrical components. The most widely used shunt circuit is that consisting of an inductance and a resistance [4] wired in either series or parallel and then connected to a piezoelectric element (Fig 1.2) that acting like a capacitance. Then, they create a RLC circuit which has dynamical characteristics similar to mass-spring-damper system. As the structure vibrates, mechanical energy is converted to electrical energy by the piezoelectric element. This electrical energy is used to drive a current through the resistor and the inductor of the shunt circuit. Optimal values of resistance and inductance can be found in order to increase the amount of piezoelectric damping to the structural mode of interest [8]. If
the resonance frequency of the circuit is tuned to some frequency value, the circuit will
draw a large value of current from the piezoelectric patch at that frequency. This current
will be dissipated in the resistance in the form of thermal energy; thus, the
electromechanical system loses some of its energy through that dissipation process.

Figure 1-2: The principle of passive control, Dissipation circuits: (a) structure series [9]
and (b) structure parallel [10].

Cady [11] introduced the concept of using the piezoelectric material as an element of
an electric circuit for the radio applications. Lesieutre [12] presented a classification of
the shunted circuits into inductive, resistive, capacitive, and switched circuits. He
emphasized that the inductive circuits which include an inductance and a resistance in
parallel with the piezoelectric-capacitor (Fig. 1-3) are the most widely used circuits in
damping as they are analogous to the mechanical vibration absorber.

Figure 1-3: Configuration of the different shunt circuits.

Hagood and von Flotow [9] introduced a non-dimensional model for piezoelectric-
shunting that indicates the damping effect of shunted circuit is similar to viscoelastic
materials. They verified their model experimentally by applying it to a cantilever beam.
They are also established the optimal analytical values of dissipation circuit elements (R, L) allowing maximum decrease of mechanical energy. Many papers [10, 13, 14] are investigated the use of passively shunted piezoelectric patches for improvement of vibration damping using the technique introduced earlier by Hagood and von Flotow [9]. Law et al. [15] presented a model based on the energy conversion rather than the mechanical approach that describes the behaviour of the material as a function of the stiffness. Two equivalent models are proposed including: an electrical model (resistance, capacitance, electric sources), and a mechanical model (force, spring, damper). A two-degree of freedom experiment was set up to test the accuracy of the model, and the experimental results were in good agreement with the predictions of the model. Tsai and Wang [16, 17] applied the combination of active and passive control simultaneously to damp the vibration of a beam using piezoelectric materials (Fig. 1-4). They presented an analytical formulation for their system and concluded that in addition to passive damping the passive shunt enhance the active control authority around the tuned frequency.

![Piezo layers](image)

Figure 1-4: A sketch of hybrid control for a cantilever beam.

The using of shunted circuits was also investigated for multimodal vibration damping. Hollkamp [18] extend the analysis of single mode damping formulation to cover multimodal damping by introducing extra circuits in parallel to the original shunt circuit. Wu [19] also investigated the multimodal damping using a different configuration of shunt circuits (Fig. 1-5). Furthermore, Wu and Bicos established these passive techniques to the control of several modes on composite plate (epoxy/fibre of glass) [20].
Recently, different attempts for broadband vibration attenuation were introduced using “Negative-Capacitance” shunts circuits [21] and the shunt circuit that built with a reactance neutralizing circuit, which is designed with operational amplifier circuits to maximize the parallel reactance of the shunt circuit over a wide frequency band [22]. Also experimental and analytical study was performed to investigate the effect of using passive shunt circuits for the control of flow induced vibration of turbomachine blades [23]. The study concluded the effectiveness of that technique in the attenuation of blade vibration. Zhang et al. [24] presented another application for shunted piezoelectric material to damping the acoustic reflections from a rigid surface. By using a one-dimensional model they concluded that it is a promising application. Warkentin and Hagood [25] investigated the use of diode and variable resistance elements in the shunt circuits to enhance the shunt circuit sensitivity. They concluded that this model improve significantly the performance of the conventional piezoelectric shunts for structural damping. Davis and Lesieutre [26] used a modal strain energy approach to predict the damping generated by shunted resistance. They introduced a variable that measures the contribution of the circuit to the energy dissipation. This variable depends on the strain induced in the piezoelectric material. Then, they applied the finite element method to determine the effective strain energy. Finally, they presented their results in terms of the conventional loss factor and confirmed their results experimentally. Saravanos [27] presented an analytical solution of the plate vibration with embedded piezoelectric elements shunted to resistance circuit. He used the Ritz method to solve the resulting coupled electromechanical equations. Park and Inman [28] compared the results of shunting the piezoelectric elements with an R-L circuit connected either in parallel or in
Series. They developed an analytical model to predict the behaviour of a beam with a shunted circuit. The predictions of the model are verified experimentally. They noted that the amount of energy dissipated in the series shunting case is directly dependent on the shunting resistance, while in the parallel case, the energy dissipated depends on the inductance and capacitance as well.

Recently, Caruso [29] presented a comparative theoretical and experimental study of different shunt circuits. Edberg et al. proposed the simulated inductance by the circuit in the type of “gyrators” [30] in order to eliminate the problem of the inductance volume in the range of low frequencies. Davis and Lesieutre [31] studied the application of the active auto regulators circuit for adaptation of resonance frequency of dissipative circuit with derivatives of mechanical systems. With their system that actively compatible, they can realise an improvement on damping in compared with passive shunt.

1.2.1.2. Limitations of passive techniques

While passive damping can greatly improve the damping of system, beyond their simplicity, the passive techniques possess certain number of limitations. First, for low structural frequencies, the value of inductance to the resonant shunt circuit can be quite large. A passive inductor of this size could easily preclude its use for lightweight applications (air/space, tennis rackets, etc). Secondly, the resonant shunt technique can only compatible to one structural mode. Then it is effective only when the tuned frequency of shunted net-work is suitable for the particular operating conditions. Such conditions are often variables of time and sensitive to environmental changes, and, hence, passive vibration absorbers and the resonant circuit are often mistuned. This can drastically reduce the effectiveness of such absorbers and the resonant shunt technique. Then the frequency of this shunted net-work should be tuneable and adaptable to the frequency variations of controlled system. For multi-modal damping, other complex shunt circuits must be used [20, 32, 33]. Then in order to provide adequate damping, different piezoelectric control must be chosen which often complicates analysis and design of the system. While piezoelectric element is easy to apply, the damping is of limited bandwidth. Alternatives to passive vibration control include semi-passive, semi-active and active control strategies.
1.2.2. Active control

1.2.2.1. Active control and its principle

This type of control is usually more performing than passive technique. But the fundamental difference is on the need of the external source of energy for driving the control actuators. Nevertheless, they are complex for practical implementation and in general rely on a complete chain comprising sensors, calculation unit, power amplifiers and actuators. The raw principle of active control is using external source to generate forces that interfere with the wave or vibration of system in order to minimise it. This method needs a net-work of sensors along with a good model of the structure to get a situation of the vibration or acoustic and actuators to act on vibration or the acoustic field. These diverse elements are controlled by a advanced system of management (microcontroller, DSP…) that calculate the optimal control permitting the minimisation of wave or vibration of system. The performance of active control is clearly relying on the algorithm of control. The concept of acoustics active control is to produce a signal identical in amplitude and opposite in phase (180 differences) to the offending noise (signal), so that the combination of two fields leads to reduction of amplitude. The control or secondary signal is produced by one or more secondary sources. On the other word, the fundamental concept of the noise reduction method (active technique) was to use a feedforward control approach in which a signal correlated with the disturbance is detected, shifted 180 in phase, and used to cancel the offending noise via a secondary excitation source. This feedforward method, still prevalent in modern active control work.

In order to adapt these techniques to complex structures with many system parameters a high performance calculator (microcontroller, DSP carte, and microprocessor) is needed. Already, these active techniques are usually difficult to implement practically due to their volume, mass and required energy. But with recent development of fast inexpensive digital signal processors, high efficiency switching amplifier the development of practical systems becomes feasible. Good overviews of the history of the active control field have been presented by Warnaka [34] and Elliot and Nelson [35]. In addition, recent textbooks by Nelson and Elliot [36] and Fuller, et al. [37] give a view in detail of the current state of the art of the active control field.
Figure 1-6 illustrates the principal functioning of the active control through this example.

![Diagram of active control system]

Figure 1-6: Schematic of control active: the control active in the car [38].

The early application of piezoelectric materials to the control vibration was developed by Olson [39]. However, due to the lack of power of the actuators, the control of vibration was limited. Recently, a great deal of interest has developed in the area of active vibration control, as described in a survey paper by Rao and Sunar [40]. Crawley and de Luis [41] provided a detailed model of the use of piezoelectric actuators to vibration control in structures. Hagood et al. [42] developed a general model which couples the effect of the piezoelectric elements with the elastic structure. Kim and Jones [43] derive a generalized formulation based on Love’s equations of motion for a composite structure and used these equations to estimate the optimal thickness of the piezoelectric actuator layers. The composite structure consists of many layers of active (piezoelectric) and non-active (bonding and structure) materials. Masters and Jones [44] addressed the optimal location and thickness of twin piezoelectric actuators embedded in a composite structure. They reported the use of a thin actuator if it is attached to the surface, or a thick actuator if it is embedded. Devasia et al. [45] found the optimal placement and size of a piezoelectric element on a beam. Recently, Zhou, Lianc, and Rogers [46] made analytical modelling of a plate with piezoelectric actuators. The influence of dimensions of these actuators was studied and experimental measurements were carried out in order to validate the presented analytical model.
Among of active control technique, the modal control method shows good performance for controlling some vibrational modes \[47, 48, 49\]. Gaudiller et al. showed that modal control is well adapted for reducing operative energy by using nonlinear modal control algorithms \[50\] for removing restored potential energy \[51\] while being adapted to complex smart structures via modal adaptive algorithm \[52\].

1.2.2.2. Limitations

Active control greatly improves the performance of control. But, the global control systems are complex and expensive. In order to implement these techniques, it is necessary to have a complex algorithm of control, as well as the software materials. In addition, there are some problems such as: First of all, in the case of failure of one of the active components (sensor, actuator or electronic failures), the structure may not have enough damping. Also, in order to adequately control the vibration, the actuator may need a large amplifier and high power input. This increases the complexity in the system and the possibility of instability \[53\]. The two main limitations of this method are that it requires external power sources to allow energy exchanges between the actuators and the structure and that a large number of components are necessary. The active control efficiency can be limited by the actuation capabilities. In the case of smart structures, the voltage applied to the actuator can be limited by the amplifier capabilities.

1.2.3. Resulted techniques from the passive and active techniques

1.2.3.1. Hybrid active-passive techniques

The objective of these techniques is to reach trade off between passive and active techniques in order to obtain good performance of control on a large band frequency by using absorbing materials (viscoelastics or piezoelectric materials) along with an active control device. One example of this method of control was studied by Lee et al. \[54\] and numbers of papers have been published in the field of vibration damping using hybrid techniques.
1.2.3.2. Semi-active techniques

A semi-active technique achieves vibration control by changing its dynamic parameters, such as stiffness or damping. The advantage of semi-active control is far less energy requirement, low costs, and lower complexity compared to active systems. The state-switching is more closely related to semi-active control concepts since only the system’s dynamic parameters are modified with no active forcing actuator. Parametric excitation generates a secondary harmonic excitation by varying the coefficients of the equations of motion. Davis et al. [55] recognized that the piezoceramic device developed by Dosch et al. [56] could be considered as a controllable variable stiffness element, due to the dependence of the piezoceramic’s stiffness on the output impedance. Clark [57] has considered piezoelectric devices for state-switching vibration control. The switching criteria used allow for switching at maximum displacement to optimize the dissipation of energy. However, switching at maximum strain will cause undesirable mechanical transients in the system [58-61]. Mechanical transients are shocks in the response due to a sudden release or addition of energy to the system violating the conservation of energy principle. The piezoelectric configuration used by Larson et al. [58-60] is similar to the approach used by Davis and Lesieutre, [26, 53] except Davis and Lesieutre’s device was implemented for vibration control, whereas Larson’s objective was underwater transduction. The stiffness of a piezoelectric element is different when it is short circuited as compared to when it is open circuited [26, 55]. If the instantaneous change in stiffness occurs at zero strain and then no mechanical transients will be introduced to the system. Piezoelectric elements have a relatively high stiffness, which can lead to relatively large absorber masses. Also, piezoelectric materials should not be put into tension, as they are fragile and have displacement amplitude limitations.

1.2.3.2.1. State switching damping technique

State switching was presented by Larson [60]. State switching, is a semi-active variable stiffness technique in which bonded piezoelectric elements are switched from the short circuit to open circuit states. This transducer is capable of instantaneously switching between two discrete stiffness. Switching between discrete stiffness changes the resonance frequency of the device, thus increasing the effective bandwidth. The
state-switched transducer uses piezoelectric material as the spring element in the system. This technique changes the stiffness of the structure for two quarters of its vibration period, thus dissipating energy. In state switching [59, 62], the piezoelectric element is switched from an open circuit state to a short circuit state at specific times in the structure’s vibration cycle (see Fig. 1-7). This switching action has the effect of removing potential (strain) energy from the mechanical system, thus removing energy from the structural mode of interest. (In this approach the piezoelectric element is short-circuited in each maximum of displacement, then held in short-circuit during quarter of structure oscillation).

![State switching circuit](image)

Figure 1-7: State switching circuit.

Clark proposed a single degree of freedom system (Figure 1-8). This system could also be thought of as a single modal model of a multi-modal system. The system consists of a mass attached to a piezoelectric stack that is in parallel with a structural stiffness. The aim is to suppress the vibration.

He introduced a variable stiffness $K: K=K_f$ in one state (short circuit) and $K=K_o$ in other state (open circuit). $\Delta K = K_o-K_f$ is the variation induced by the control process. Where $K_o$ and $K_f$ are the equivalent open circuit and short circuit stiffness, respectively.

An excitation force with frequency equal to resonance frequency of system is applied on the mass $M$. When the mass reaches maximum displacement, the potential energy is at a maximum, defined by

$$E_{p,\text{max}} = \frac{1}{2}K_ou_M^2$$

(1.4)
At this point, the element is switched to a low stiffness state (short circuit) so that the potential energy is

\[ E_{p_{\text{max}}} = \frac{1}{2} K u_{M}^2 \]  

(1.5)

which is less than before. The difference in energy is

\[ \Delta E_{p_{\text{max}}} = \frac{1}{2} (K_2 - K_1) u_{M}^2 \]  

(1.6)

Figure 1-8: Single degree of freedom system of a mass and stacked piezoelectric element [62].

This amount of mechanical energy is released from the system when switching occurs [62]. The component is kept in the short circuit state until the modal displacement returns to equilibrium (i.e. \( u = 0 \)). Then the piezoelectric element is switched back to the open circuit state and the entire cycle repeats.

Because of the structural stiffness is in parallel with the piezoelectric element, the total change in the system’s stiffness is small; therefore, only small amounts of mechanical energy are dissipated using the state switching technique. Also note that state switching only dissipates mechanical energy over two quarters of the vibration cycle where as the following introduced technique (semi-passive) dissipate mechanical energy for approximately the entire vibration cycle.
1.2.3.3. Semi-passive control technique

To confront with different limitations cited previously the other methods are born. All of these methods rely on nonlinear behaviour of electrical voltage generated by piezoelectric inserts.

Semi-passive vibration control devices using nonlinear methods have experienced significant development in recent years, due to their performance and advantages compared with passive and active techniques. More precisely, SSD (Synchronised Switch Damping) and derived techniques, which were developed in the field of piezoelectric damping, lead to a very good trade-off between simplicity, required power supply and performance. This technique that has developed in LGEF (laboratory of electrical engineering and ferroelectricity of INSA-Lyon) consists of nonlinear processing of the piezoelectric element voltage which induces an increase in electromechanical energy conversion [4, 63-66]. It was shown that the control law consisting of triggering the inverting switch on each extremum of the voltage (or strain) was optimal for harmonic excitation. However, in the case of complex vibrations induced by either a shock or repetitive random shocks or a random excitation force applied to a structure, the previous trivial control law does not perform well. The problem encountered is that many local maxima occur and it is difficult to determine which one should be selected for triggering the switching sequence in order to maximise the piezoelectric voltage and to enhance the energy extraction and damping performance on a wide frequency range of vibrations. A new control law is proposed in this work. In this case, the voltage or displacement signal is analyzed during a given time window and the statistically probable displacement or voltage level threshold is determined from both the average and standard deviation of the signal during the observation period. The voltage step still occurs on a local maximum of the signal but only above the statistically defined threshold. A significant decrease in vibration energy is demonstrated experimentally and theoretically in the case of a clamped beam excited by random noise. It is shown that in the case of multimodal vibration, all of the modes are controlled and preferentially those corresponding to the highest displacement amplitudes. This nonlinear damping technique consists of adding a switching device in parallel with the piezoelectric elements. The current in the switching device is always
zero except during the voltage inversion that takes place at each switch trigger (a simple circuit that made up with an electronic switch and an inductance).

SSD damping with comparison to active damping techniques does not use external sources, it directly decrease the vibration energy of structure in the form of electrical energy. Nevertheless, it needs a little quantity of external electrical energy which is used only by electronic control board which can also be self-powered. Compared to passive technique, it has more effective damping in a larger band of frequency. Then these semi-passive methods are interesting alternatives rather than active and passive methods. The simplicity and efficiency of this technique allow to application in various domains: acoustic control, vibration control [67, 68] or harvesting energy (Synchronized Switch Harvesting (SSH)) [1, 65, 69].

The principle of this technique consist of connecting the piezoelectric element on a electrical circuit in particular moments, which correspond to extremums of piezoelectric voltage [4, 66]. The piezoelectric element is in open circuit except in each extremum of strain where it is short-circuited which leads to put the voltage out of phase with the vibration velocity in order to optimise the energy extracted from the piezoelectric elements. This treatment may be assimilated to dry friction. The consequence of this process is an accumulation of charge on the capacity of the piezoelectric elements. It is important to mention that Corr et Clark [70] experimented the SSD approach in the case of multimodal vibration by using a numerical treatment to define exactly the instant of switch according to the desired targeted modes.

1.3 Goal and motivation of this work

The excellent results are obtained by active control, but the system is a large consumer of energy and also needs cumbersome and costly materials.

In order to vibration control, we are going to concentrate our work on the study and development of semi-passive technique based on probabilistic and statistical moments theories to define the instant of switch that allowing to optimise the vibration damping using piezoelectric material, for broadband excitation forces. Numerical tools and
experimental set up are developed to check upon this idea. It appears that the proposed strategy work well for various kinds of excitation forces. This type of vibration control devices lead to high performances in terms of damping and robustness. It also exhibits a unique feature since the system can be self-powered. It means that no wiring (wireless system) or power supply are needed. The advantage of this method is the simplicity of its integration that can not guarantee an active approach.

In chapter two will be discussed on the SSD method and its modelling. Chapter three will discuss the semi-passive random vibration control based on statistics. Semi-passive vibration suppression performance using sliding time window are described in chapter four. The damping behaviour of piezoelectric material are discussed in chapter five as well as the effect of piezo-elements surface area on the damping in high and low values of excitation force parameters (amplitude and frequency). The effect of boundary conditions on the piezoelectric damping in the case of semi-passive vibration control technique is experimentally represented in chapter six. Finally, the general conclusions are reported in chapter seven.
Chapter 2:

Semi-passive vibration control technique

This chapter has been allocated to discuss and modelling of SSD damping technique. A multimodal model of structure is used to understand the principle of electromechanical coupling and dissipation of resulting energy.

2.1 Introduction

There are many types of vibration control based on piezoelectric materials. These materials bonded on or embedded in the structure that should be damped. It converts one part of mechanical energy to electrical energy when structure vibrates. This transfer or reduction of mechanical energy of structure results the damping of vibration. Active control is a complex system contains the sensor for vibration measurement, a calculator for deduce the applied voltage on the active piezo-elements to control of vibration, and at the final voltage generators and amplifier for exciting the actuators. In the passive techniques a passive dissipative electrical shunt circuit is connected to piezo-elements. The more effective method consist an inductance with a resistance in the series form and they are parallel with the piezo-elements capacitance. The optimum damping is obtained when the electrical resonance frequency of oscillation circuit is compatible
with the resonance frequency of structure. It is very simple rather than active technique. However, this passive technique has some drawbacks. For the low resonance frequencies, the optimal value of inductance become very large and it is necessary to use specific active circuit (gyrators). One alternative innovation in the type of semi-passive vibration control technique has been developed in the LGEF laboratory which can be autonomous but realised an active behaviour of voltage generated by piezo-elements [4, 63]. This technique called SSD (synchronised switch damping) is implemented by using an electronic switch that the piezoelectric element is briefly switched to an electrical shunt circuit that can be a simple short-circuit (SSDS), or a small inductance (SSDI) at specific times in the structure’s vibration cycle. In these cases, the nonlinear treatment of voltage creates a mechanical force in the form of piecewise function which is out of phase with the vibration velocity. This signifies that from the view of structure the nonlinear treatment introduce a dissipative mechanism such as dry friction. Moreover, in this case, it is not necessary to have any information on the modes of structure and then it is very adaptable. This chapter presents this technique.

2.2 SSD damping techniques

2.2.1. SSDI technique

The principle of the SSD technique consists of adding a switch device in parallel to the piezo-elements that provoke an inversion of the voltage in each extremum of the piezoelectric voltage or displacement. In each switch, the piezoelectric elements are connected to an oscillating circuit (SSDI) or simply short-circuited (SSDS). In the case of SSDS, this device is a simple electronic switch, and in the case of SSDI technique, the switch is in series with an inductance $L$. The switch is always open, except when an extremum of displacement or voltage is detected. In the case of SSDI, the piezo-element capacitance $C_p$ and the inductance $L$ constitute an oscillating circuit. The switch is hold close till the piezoelectric voltage is completely inverted. The inversion time $\Delta t_i$ is equal to the half period of oscillatory circuit and express by:

$$\Delta t_i = \left(2 f_s\right)^{-1} = \pi \left(C_p L\right)^{1/2}$$ (2.1)
By attention to Eq. (2.1), the inversion time is decreased with small values of the inductance. Thus in this technique it is not necessary to use the large values of inductance contrary to classic passive technique. Practically in the case of SSDI, the value of inductance is selected in this way that the period of the oscillatory circuit \((2\Delta t_s)\) is 10 to 50 times smaller than the period of mechanical vibration (or the value of \(L\) is chosen in such way that the frequency of the oscillatory circuit \(f_s\), during the switch, is 10 to 50 times of the resonance frequency of structure). When switch is open, the outgoing current from the piezo-elements is zero and the evolution of voltage is similar to deformation evolution. The switch device of SSD technique has been represented as well as their waves form in Fig. 2-1.

![Diagram](image)

Figure 2-1: SSD techniques, circuits and their waves form: (a) SSDS, (b) SSDI.

In the case of SSDI, when the piezo-elements are connected to inductance \(L\), the
natural frequency $\omega_s$ of oscillating circuit $(L, C_p)$ is:

$$\omega_s = \frac{1}{\sqrt{LC_p}}$$  \hspace{1cm} (2.2)

In this case, the voltage inversion (switch) is not perfect, because one part of the energy stored on the piezoelectric element’s capacitance is lost in the switching network (electronic switch, inductance). These losses are modelled by an electrical quality factor $Q_i$. The relationship between $Q_i$ and the voltage of the piezoelectric element before and after the inversion process is given by:

$$v_{\text{after}} = -\gamma v_{\text{before}} = -v_{\text{before}} e^{-\frac{-\pi}{2Q_i}}$$  \hspace{1cm} (2.3)

$\gamma$ is the inversion coefficient. In the case of the SSDS technique, the voltage is not inverted but simply cancelled. Then SSDS technique corresponds to the case where $\gamma=0$ and may be considered such as a particular case of SSDI technique when $Q_i=0$. In this case the piezo-elements are short-circuited in each extremum of displacement or voltage during a very short time allowing eliminating of voltage. This coefficient $\gamma$ could also be increased using external voltage sources as in the SSDV case. It can be improved as well as the corresponding damping as shown in Ref. [71]. In open circuit, the piezo-elements voltage increase proportional to deformation. Then after each inversion, the capacity $C_p$ is also charged to voltage $v_k$. In this case, the piezoelectric elements continue to their charge by piezoelectric effect. Consequently, the voltage on their borne becomes a simple image of the deformation. In the instant $t_k$ corresponding to the closed switch, the electrical charges flowing in the circuit generate current $I_{\text{out}}$. This current circulates during a very short period of time equal to $\Delta t_s$.

In the case of SSDI technique, when the switch is closed (Fig. 2-5a), the governing equation for the shunt circuit can be written as [8]

$$v_L = v_{C_p} + v_{R_i}$$

$$\dot{Q} + \frac{R_i}{L} \dot{Q} + \omega_s^2 Q = 0$$  \hspace{1cm} (2.4)

The inversion quality factor relies on $R_i$, the resistance of the switching network:
Then the shunt inductance $L$ is calculated from $\omega_s$ (resonant frequency of the oscillating circuit) and the piezo-element capacitance $C_p$.

$$L = \frac{1}{\omega_s^2 C_p} \tag{2.6}$$

The dependent variable in Eq. (2.4), $Q$, is the outgoing charge from the piezoelectric elements to the shunt circuit. The key to achieving energy dissipation in the mechanical system is to properly phase of piezo-elements voltage and the mechanical motion. The response of $Q$ in Eq. (2.4) will be oscillatory while the switch is closed, but will increase similar to deformation if the switch is reopened. So, the approach to the SSD techniques is to close the switch when the mechanical system reaches extremum displacement (thus providing maximum charge to the shunt), and then reopening the switch when $Q$ reaches a peak that is out of phase with the modal velocity over the next half-cycle, and the system is dissipative [4, 63, 72]. With this technique, then, there are only three parameters that need to be determined for the RL shunt: the optimal resistance, length of time the switch should be shut, and optimal inductance. One of the factors that determine the amount of outgoing charge from the piezoelectric elements is the amount of resistance in the RL shunt. The lower resistance in the shunt, result the larger amount of charge that can be built up and outgoing from the elements. Therefore, in order to build up the maximum amount of outgoing charge from the piezoelectric elements, the resistance in the circuit must be kept as low as possible. So, the only resistance in the RL shunt circuit should be the resistance of the inductor, the switch, and the adjoining wires. Once the switch is shut (at maximum charge), it should be reopened as the charge in the RL shunt circuit reaches a peak that is opposite in sign to that which it started (other consequent maximum). This ensures that the piezo-elements voltage has been completely inverted; therefore, maximizing its effect upon the system. This occurs approximately at $1/2$ of the period of the electrical circuit. Since the resistance in the RL shunt is designed to be very small, it can be ignored in the calculation of the electrical period. Therefore, the time period the switch should be shut is $\Delta t_s$ (Eq. 2.1).
Since the RL shunt for the SSDI technique is not tuned to a specific mechanical resonant frequency, the inductor of the shunt circuit can be any value. However, an upper and a lower bound on this inductance should be set to optimize SSDI performance. An upper bound for the inductance is set to ensure that the electrical natural frequency is at least ten times faster than the mechanical mode of interest. This upper limit is set to ensure that the charge generated by the elements remains approximately constant while the shunt switch is shut. A lower bound for the shunt inductance is quite arbitrary but is strongly related to the switching losses. However, a small shunt inductor causes a high electrical natural frequency, which leads to a rapid build up of outgoing charge. This has the effect of applying step inputs to the system. This type of input can excite high-frequency modes of the system and induce ‘chatter’. To avoid this, a lower bound on the shunt inductor should be set such that the electrical natural frequency is not more than 50 times faster than the mechanical mode of interest. For a detailed description of the SSDI energy reduction methods and procedures, see Corr and Clark [72].

The force due to the piezoelectric patches [8] can be derived by examining the one-dimensional loading piezoelectric constitutive equations or standard piezoelectric Eqs. (2.7) and (2.8) [19].

\[
T_s = E^s S - eE \quad (2.7)
\]
\[
D = eS + e^s E \quad (2.8)
\]

Noting that:

\[
v = -L_p \ddot{E} \quad , \quad Q = AD \quad , \quad I_{out} = \dot{Q} \quad , \quad S = \frac{u_p}{L_p} \quad , \quad T_s = \frac{F_p}{A}
\]

Substituting Eq. (2.9) into Eqs.(2.7) and (2.8) yields

\[
F_p = K_{pe} u_p + \alpha v \quad (2.10)
\]
\[
Q = \alpha u_p - C_p \dot{v} \quad \text{or} \quad I_{out} = \alpha \dot{u}_p - C_p \ddot{v} \quad (2.11)
\]

where \( \alpha = \frac{eA}{L_p} \quad ; \quad K_{pe} = \frac{E^s A}{L_p} \quad ; \quad C_p = \frac{e^s A}{L_p} \).
where \( L_p \) is the thickness of the piezo-element (m), \( A \) is the total surface area of the piezo-element (m\(^2\)), \( T_s \) stress (N/m\(^2\)), \( S \) strain, \( D \) electric displacement (C/m\(^2\)), \( \varepsilon^S \) is the permittivity of piezo-element under constant strain (F/m), \( E^F \) young’s modulus under constant electric field (N/m\(^2\)), \( e \) is the piezo-element constant (C/m\(^2\)), \( I_{out} \) outgoing current from the piezo-element (A), \( F_p \) is the force due to the piezoelectric elements (N) (the reaction force of the piezoelectric element), \( \alpha \) is the electrically dependent part of the force applying by the piezo-element on the structure (piezo-effect force) and \( K_{pe}u_p \) is the elastic force), \( Q \) is the outgoing electrical charge(C); \( u_p \) is the piezoelectric deformation (m) and \( \alpha \) is the force factor (N/V), that is the ratio between the force \( F_p \) and the voltage \( v \) in the zero deformation. \( \bar{E} \) electric field (V/m), \( K_{pe} \) short-circuit equivalent piezoelectric stiffness, \( C_p \) is the constant strain piezoelectric capacitance (F).

The electromechanical coupling coefficient \( k \) can be defined as \([4, 8]\)

\[
k^2 = \frac{\text{energy converted}}{\text{input energy}}
\]

\[
k^2 = \frac{\alpha^2}{K_{pe}C_p + \alpha^2} = \frac{\alpha^2}{K_{de}C_p}
\]

(2.12)

\( K_{de} \) is the equivalent open-circuit piezoelectric stiffness \((K_{de} = \frac{K_{pe}}{1-k^2})\)

2.2.2. The nonlinearity of SSD technique

If the excitation force is sinusoidal with the frequency \( \omega \), the displacement \( u \) remains sinusoidal and it can be explained in the form of Eq. (2.14). Where \( u_M \) is the amplitude of displacement. In this case the current \( I_{out} \) in the switching condition is defined by Eq. (2.11). In open circuit condition \( I_{out} = 0 \), then from this equation.

\[
v(t) = \frac{\alpha}{C_p}u(t) + v_{const}
\]

(2.13)

where \( v_{const} \) is a constant that defined over an interval between two consequent switches. It is the control voltage corresponds to dry friction. As shown in the Fig. 2-2, the effect of SSD control define \( v_{const} \) as a piecewise function (while without any control \( v_{const} = 0 \) due to continuity and zero initial conditions). Thus, the resulting voltage of piezo-
elements can be explained as the sum of two functions, the first is an image of deformation (with proportional coefficient $\alpha/C_p$). It generates a non dissipative force and affects the stiffness of structure. The other is a piecewise function in out of phase with the velocity. This has been explained by Eq. (2.15) that $h$ is a piecewise function. In open circuit (Between two consequent voltage inversions), the voltage variations are proportional to displacement with factor $\alpha/C_p$, whereas, there is no displacement variations during the voltage inversions, since the inversion time is much smaller than the mechanical motion period.

$$u(t) = u_M \sin(\omega t + \theta)$$  \hspace{1cm} (2.14)

$$v = \frac{\alpha}{C_p} (u + h)$$  \hspace{1cm} (2.15)

Function $h$ can be calculated from Eq. 2-2. We well have the set of Eqs. (2.16) from Eq. (2.15).

$$\begin{align*}
    v_k &= \frac{\alpha}{C_p} (u_k + h_k) \\
    v_{k+1} &= \frac{\alpha}{C_p} (u_k + h_{k+1})
\end{align*}$$  \hspace{1cm} (2.16)

Where $v_k$ and $v_{k+1}$ are the values of voltage before and after the inversion and $u_k$ is the value of displacement at the instant of switch ($t_k$). From these equations and Eq. (2.3), the expression of function $h$ can be deduced.

$$h_{k+1} = -\gamma h_k - (1 + \gamma)u_k$$  \hspace{1cm} (2.17)

In the permanent regime $h_{k+1} = -h_k$ then:

$$h_k = \frac{1+\gamma}{1-\gamma} u_k$$  \hspace{1cm} (2.18)

Function $h(t)$ is a piecewise function that its sign is opposite to sign of velocity between two instants $t_k$ and $t_{k+1}$. It can then write as:

$$h_k(t) = -\frac{1+\gamma}{1-\gamma} u_M \text{sign}(\ddot{u}(t))$$  \hspace{1cm} (2.19)

The voltage inversion in the SSDI technique increases the piecewise function amplitude and then the efficiency of this technique. The nonlinear treatment of piezo-
voltage is clearly visible in Fig. 2-2. The SSDS results can be deduced from the SSDI with \( \gamma = 0 \).

![Diagram showing voltage, velocity, and displacement]

Figure 2-2: Decomposition of SSDI voltage.

### 2.3 Theoretical model of the semi-passive piezoelectric device

#### 2.3.1. Multimodal structure modelling

A multi-modal model is proposed for modelling of vibrating cantilever beam (Fig. 2-3) that the piezoelectric elements have been bonded on the clamped-end of beam. This model is compared to a real structure. The piezo-elements have been made with massive ceramics polarized perpendicular to beam. Then the lateral coupling \( k_{31} \) is predominant.

The piezo-elements have been connected to each other in parallel. The electrical ground is the electrodes that connected to the beam.

The differential equation for a beam with lateral motion is given by Eq. (2.20) [73] where \( u(x,t) \) is the lateral beam deflection according to Fig. 2-3.
Figure 2-3: Schematic diagram of cantilever beam. \( u(x,t) \) is the beam deflection along the transverse direction \((y)\) and \( f(t) \) is the excitation force at the tip of the beam \((x=L)\).

\( E \) and \( J \) represent Young’s modulus and the inertia moment of the cross-section, and \( \rho \), \( c \) and \( f(x,t) \) are the mass, damping and excitation force per unit length, respectively. This equation corresponds to Euler-Bernoulli assumptions.

\[
\rho \frac{\partial^2 u(x,t)}{\partial t^2} + c \frac{\partial u(x,t)}{\partial t} + \frac{\partial^2}{\partial x^2} \left[ EJ \frac{\partial^2 u(x,t)}{\partial x^2} \right] = f(x,t)
\]  

(2.20)

It is interesting to find the particular solution of Eq. (2.20) for a general excitation force per unit length. First, we assume that the excitation force is given by

\[
f(x,t) = p(x)f(t)
\]

(2.21)

where \( p(x) \) is the load’s spatial distribution and \( f(t) \) is the input time history. Second, we assume that the particular solution is given by

\[
u(x,t) = \sum_{i=1}^{N} \phi_i(x) q_i(t)
\]

(2.22)

where \( q_i(t) \) and \( \phi_i(x) \) are the modal coordinates and the eigenmode functions, respectively. \( N \) is the number of modes. The eigenmode functions are also orthogonal thus satisfying the relations

\[
\int_{0}^{l} \rho \phi_i(x) \phi_k(x) \, dx = \begin{cases} 0 & \text{for } i \neq k \\ M_i & \text{for } i = k \end{cases}
\]

(2.23)

\[
\int_{0}^{l} \frac{d^2}{dx^2} \left[ EJ \frac{d^2}{dx^2} \phi_i(x) \right] \phi_k(x) \, dx = \begin{cases} 0 & \text{for } i \neq k \\ K_i & \text{for } i = k \end{cases}
\]

(2.24)

where \( M_i \) and \( K_i \) are the generalised mass and generalised stiffness for the \( i \)th mode, respectively, and \( l \) is the cantilever beam length. After insertion of Eqs. (2.21) and (2.22) into Eq. (2.20) and multiplying it by \( \phi_k(x) dx \), integrating on length \( l \) and
employing the orthogonality condition of Eqs. (2.23) and (2.24), along with assuming that damping is proportional to the mass and stiffness distributions, the result is

$$M_i \ddot{q}_i + c_i \dot{q}_i + K_i q_i = \overline{Q}_i f(t) \quad (2.25)$$

where $q_i(t)$ can be determined from these equations. $c_i$ is the $i$th generalised damping coefficient and $\overline{Q}_i$ is the generalised excitation force [73] given by

$$\overline{Q}_i = \int_0^l p(x) \phi_i(x) dx \quad (2.26)$$

For the cantilever beam problem represented in Fig. 2-3, the concentrated external force $f(t)$ is applied at the free-end of the beam and the piezoelectric patches are bonded on the structure Eq. (2.25) becomes

$$M_i \ddot{q}_i + c_i \dot{q}_i + K_{ei} q_i = \varphi_i(l) f(t) - \alpha_i \nu_i(t) \quad (2.27)$$

where $K_{ei}$ is the short-circuit generalised stiffness. The term $\alpha_i \nu_i(t)$ corresponds to the force due to $i$th component of the voltage according to the macroscopic piezoelectric coefficient $\alpha_i$. These coefficients depend on the modes considered as well as the corresponding stress distributions and introduce the electromechanical coupling.

Also, Eq (2.27) can be calculated for the modal coordinates $q_i(t)$ from the Lagrange’s equations [74-76] by first establishing the kinetic, potential and dissipation energies. Recognizing the orthogonality relation (Eqs. 2.23, 2.24) the kinetic energy $E_k$ can be defined as:

$$E_k = \frac{1}{2} \int_0^l \dot{u}_i^2(x,t) \rho(x) dx = \frac{1}{2} \sum_i M_i \dot{q}_i^2(t) \quad (2.28)$$

similarly the potential energy is:

$$E_p = \frac{1}{2} \int_0^l \text{EJ}[u''(x,t)]^2 dx = \frac{1}{2} \sum_i K_i q_i^2(t) = -\frac{1}{2} \sum_i \omega_i^2 M_i q_i^2 \quad (2.29)$$

where the generalized stiffness $K_i$ is:

$$K_i = \int_0^l \text{EJ}[\varphi_i''(x)]^2 dx \quad (2.30)$$

the dissipation energy $E_d$ [74] is defined as:

$$E_d = \frac{1}{2} \sum_i c_i \dot{q}_i^2(t) \quad (2.31)$$
where the generalized damping coefficient $c_i$ is defined as:

$$c_i = \int_0^l c(x) \varphi_i^2(x) \, dx \quad (2.32)$$

as long as proportional damping can be assumed as represented by Eq. (2.33) where $c(x)$ is the damping per unit length.

$$\int_0^l c(x) \varphi_i(x) \varphi_k(x) \, dx = 0 \quad \text{for} \quad i \neq k \quad (2.33)$$

the work of the applied external force along a virtual displacement $\delta q_i$ is:

$$\delta w = \sum_i \delta q_i \int_0^l f(x,t) \, \varphi_i(x) \, dx = \sum_i \overline{Q}_i f(t) \delta q_i \quad (2.34)$$

substituting into Lagrange’s equation leads to:

$$\frac{d}{dt} \left( \frac{\partial E_k}{\partial \dot{q}_i} \right) - \frac{\partial E_k}{\partial q_i} + \frac{\partial E_p}{\partial \dot{q}_i} + \frac{\partial E_d}{\partial \dot{q}_i} = \overline{Q}_i f(t) \quad i = 1,2,\ldots,N \quad (2.35)$$

where N is the number of considered significant modes. The differential equations for the $q_i(t)$ are a set of independent equations:

$$\ddot{q}_i + 2\zeta_i \omega_i \dot{q}_i + \omega_i^2 q_i = \overline{Q}_i f(t) \left( \frac{M_i}{E_i} \right) \quad (2.36)$$

In these equations, $\zeta_i$ are the damping ratios. For the cantilever beam problem represented in Fig. 2-3, Eq. (2.36) become

$$\ddot{q}_i + 2\zeta_i \omega_i \dot{q}_i + \omega_i^2 q_i = \frac{1}{M_i} f(t) \varphi_i(l) - \frac{\alpha_i}{M_i} v_i(t)$$

$$M_i \ddot{q}_i + c_i \dot{q}_i + K_i q_i = \varphi_i(l) f(t) - \alpha_i v_i(t) \quad (2.37)$$

where the generalized forces are $\varphi_i(l)$ [77]. Three modes shape of the beam have been shown in Fig. 2-4 that normalised rather than the free-end displacement of the beam.
Figure 2-4: modes shape of the structure.

In the open circuit condition, the piezoelectric voltage $v_i$ due to the $i$th mode of vibration is:

$$v_i = \frac{\alpha_i}{C_p} q_i(t)$$  \hspace{1cm} (2.38)

The global piezoelectric voltage ($v$) and the total outgoing current ($I_{out}$) from the piezoelectric patches resulting from the modes superposition are:

$$v = \frac{1}{C_p} \sum_{i=1}^{N} \alpha_i q_i(t)$$  \hspace{1cm} (2.39)

$$I_{out} = \sum_{i=1}^{N} \alpha_i \dot{q}_i - C_p \dot{v}$$  \hspace{1cm} (2.40)

In these equations, $C_p$ is the capacitance of the piezoelectric elements and $v$ is the total piezoelectric voltage. In general, the piezoelectric patches are wired together in parallel. The open-circuit stiffness $K_{di}$ of the structure is related to $K_{ei}$, $\alpha_i$ and $C_p$ by:

$$K_{di} = K_{ei} + \frac{\alpha_i^2}{C_p}$$  \hspace{1cm} (2.41)

The eigenmode angular frequencies $\omega_{di}$ in open-circuit and $\omega_{ei}$ in short-circuit are defined as:

$$\omega_{di} = \sqrt{\frac{K_{di}}{M_i}}; \hspace{1cm} \omega_{ei} = \sqrt{\frac{K_{ei}}{M_i}}$$  \hspace{1cm} (2.42)

The relationship of coupling coefficients $k_i$, mechanical quality factor $Q_{mi}$, $\alpha_i$ and $K_{ei}$ with other parameters are defined as:
The various parameters previously defined can be identified (Table 2-1).

Table 2-1: Modal parameters of the piezoelectric structure.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_{di}$</td>
<td>Open-circuit angular frequency for mode $i$</td>
<td>Measured</td>
</tr>
<tr>
<td>$\omega_{ei}$</td>
<td>Short-circuit angular frequency for mode $i$</td>
<td>Measured</td>
</tr>
<tr>
<td>$Q_{mi}$</td>
<td>Mechanical quality factor for mode $i$</td>
<td>Measured</td>
</tr>
<tr>
<td>$\lambda_i$</td>
<td>Proportionality coefficient between open circuit voltage $v$ and beam tip deflection</td>
<td>Measured</td>
</tr>
<tr>
<td>$C_p$</td>
<td>Piezoelectric blocked capacitance</td>
<td>Measured</td>
</tr>
<tr>
<td>$M_i$</td>
<td>Modal mass for mode $i$</td>
<td>Computed from Eq. (2.23)</td>
</tr>
<tr>
<td>$K_{di}$</td>
<td>Open-circuit stiffness for mode $i$</td>
<td>Computed from Eq. (2.42)</td>
</tr>
<tr>
<td>$K_{ei}$</td>
<td>Short-circuit stiffness for mode $i$</td>
<td>Computed from Eq. (2.42)</td>
</tr>
<tr>
<td>$c_i$</td>
<td>Damping coefficient for mode $i$</td>
<td>Computed from Eq. (2.42)</td>
</tr>
<tr>
<td>$\alpha_i$</td>
<td>Macroscopic piezoelectric coefficient for mode $i$</td>
<td>Computed from Eq. (2.43)</td>
</tr>
</tbody>
</table>

The response of model to an excitation $f(t)$ may be calculated by numerical integration of the constitutive Eqs. (2.27) and (2.40). This model has added a degree of freedom to the system which is the voltage of the piezoelectric patches.

### 2.3.2. Energy balance and energetic consideration/SSD technique

In the proposed control technique, an electric circuit is connected to the piezoelectric elements in order to perform vibration damping or harvest energy. Multiplying each term of Eq. (2.27) by $\dot{q}_i$, integrating over the time and summing the various modes lead to energy balance of the system. This energy balance is summarised by

$$ E_f = E_m + E_d + E_t $$

(2.44)
with the various energy terms detailed in Table 2-2. The energy supplied, $E_f$, by the external force is distributed as mechanical energy $E_m$, viscous losses $E_d$ and transferred energy $E_t$. This transferred energy corresponds to the part converted into electrical energy. The goal is to maximise this energy. It is therefore necessary to establish methods to define the accurate switch trigger time that would maximise the electric power produced by the piezoelectric elements. Multiplying each of the terms of Eq. (2.40) by the voltage and integrating over the time shows that the transferred energy is the sum of the electrostatic energy stored on the piezoelectric elements and the energy absorbed or dissipated by the electrical device (Table 2-2).

Table 2-2: Structure energy definitions.

<table>
<thead>
<tr>
<th>Energy</th>
<th>Definitions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supplied energy</td>
<td>$E_f = \sum_{i=1}^{N} \varphi_i(t) \int_{0}^{t} f(t)q_i(t)dt$</td>
</tr>
<tr>
<td>Mechanical energy</td>
<td>$E_m = \frac{1}{2} \sum_{i=1}^{N} M_i q_i^2 + \frac{1}{2} \sum_{i=1}^{N} K_{ei} q_i^2$</td>
</tr>
<tr>
<td>Viscous losses</td>
<td>$E_d = \sum_{i=1}^{N} c_i \dot{q}_i^2 dt$</td>
</tr>
<tr>
<td>Transferred energy</td>
<td>$E_t = \sum_{i=1}^{N} \alpha_i \int_{0}^{t} \dot{q}<em>i v_i dt = \frac{1}{2} C_p v^2 + \int</em>{0}^{t} v I_{out} dt$</td>
</tr>
<tr>
<td>Kinetic energy</td>
<td>$E_k = \frac{1}{2} \sum_{i=1}^{N} M_i \dot{q}_i^2$</td>
</tr>
<tr>
<td>Potential energy</td>
<td>$E_p = \frac{1}{2} C_p v^2 + \frac{1}{2} \sum_{i=1}^{N} K_{ei} q_i^2$</td>
</tr>
<tr>
<td>Dissipated energy</td>
<td>$E_{dp} = \sum_{i=1}^{N} c_i \dot{q}<em>i^2 dt + \int</em>{0}^{t} v I_{out} dt$</td>
</tr>
</tbody>
</table>

For vibration damping, or extracted energy the electrical device connected to the piezoelectric elements is designed to have a maximize consumption. SSD techniques have been designed for this purpose. Then, another energetic equation (Eq. 2.45) can be given, expressing the supplied energy as the sum of the kinetic energy $E_k$, potential energy $E_p$ (electrostatic + elastic) and the dissipated energy $E_{dp}$ (viscous losses + energy dissipated in the electrical device). These energies are detailed in Table 2-2.

$$E_f = E_k + E_p + E_{dp}$$  \hspace{1cm} (2.45)
Table 2-3 show the summary of the extracted energy expressions according to the various switching types [1]. It is evident that the optimisation of the SSD technique (maximising the energy extracted by the switching device) is obtained by maximising the sum of the piezoelectric voltage squared before each switch. It should be noted that the value of $v_k$ in the case of SSDI is more than the one in SSDS (because of inductance).

<table>
<thead>
<tr>
<th>Utilised techniques</th>
<th>extracted energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSDS</td>
<td>$\frac{1}{2} C_p \sum v_k^2$</td>
</tr>
<tr>
<td>SSDI</td>
<td>$\frac{1}{2} C_p \sum v_k^2 (1-\gamma^2)$</td>
</tr>
</tbody>
</table>

If integral time is done during one period in the case of harmonic oscillation (steady–state forced harmonic vibration), the kinetic and potential energy terms in Eq. (2.45) are equal to zero and consequently this equation leads to

$$E_f = E_{dp}$$

(2.46)

This explains the supplied energy (the work done by external force) in the harmonic oscillations during one period is equal to dissipated energy in that period [78].

2.3.3. SSD switching device model

This nonlinear damping technique consists of adding a switching device in parallel with the piezoelectric elements. The current in the switching device is always zero except during the voltage inversion that takes place at each switch trigger. At each inversion, the energy extracted from the piezo-element is equal to the difference in the electrostatic energy on the piezoelectric elements before and after the voltage inversion jump. The energy dissipated in the switching device is then given by

$$\int_0^t v_l v_o dt = \frac{1}{2} C_p \sum_{k} v_k^2 (1-\gamma^2)$$

(2.47)

where $v_k$ is the piezoelectric voltage just before the $k$th inversion and $\gamma$ is the inversion coefficient. In the following numerical simulations, the switching sequence is simply modelled at the switch triggering time by Eq. (2.3). The voltage of piezo-element can be
calculated during the switching time from the governing equations of the shunted circuit as the following. From Fig. 2-5a for the short-circuit, we have:

\[ v = v_L + v_R = L \dot{I}_{out} + R_i I_{out} \]  

(2.48)

By substituting of Eq. (2.40) into (2.48)

\[ \dot{v} = \frac{1}{C_p} \sum_{i=1}^{N} \alpha_i \dot{q}_i + \frac{R_i}{L} \sum_{i=1}^{N} \alpha_i \dot{q}_i - \frac{R_i}{LC_p} \dot{v} - \frac{v}{LC_p} \]  

(Short-circuit)  

(2.49)

From Fig. 2-5b for the open-circuit, we have:

\[ \dot{v} = \frac{1}{C_p} \sum_{i=1}^{N} \alpha_i \dot{q}_i - \frac{v}{RC_p} \]  

(Open-circuit)  

(2.50)

Where \( R \) is the loading resistance and \( R_i \) is the inductance resistance.

\[ \sum_{i=1}^{N} \alpha_i \dot{q}_i \]

(a)

\[ \sum_{i=1}^{N} \alpha_i \dot{q}_i \]

(b)

Figure 2-5: Schematic of SSDI shunt circuit; a) short-circuit b) open-circuit.

2.4 Semi-Active vibration control

The semi-passive damping technique mentioned above is based on increasing the energy of conversion cycles by nonlinear treatment of generated voltage by the piezoelectric elements. The SSDI technique that consists of voltage inversion in each
extremum of deformation is more effective than the SSDS technique. In order to improve the performance of these nonlinear techniques, it can be increased more again the conversion cycles by inverting the voltage around a constant electrical potential. This technique is called SSDV for Synchronized Switch Damping on Voltage source. It is qualified the semi-active technique, since contrary to SSDS and SSDI techniques, an external source of continues voltage is necessary, but contrary to active techniques, the power amplifier and complex algorithm is not necessary. This Section presents the principle of this technique and its modification.

2.4.1. Principle of original SSDV

The amplitude of inverted voltage and consequently the dissipated energy are increased by using a constant potential at the instant of switch. The electrical circuit of the original SSDV technique has been represented in the Fig. 2-6a. The difference with compared to SSDI technique is that the piezoelectric element is switched on a positive or negative voltage source across the inductive shunt circuit $L$. The voltage does not inverted rather than a zero potential, but inverted rather than a constant potential, that is for increasing the effect of electromechanical conversion cycle. The control strategy of the electronic switches consists in closing $SW_1$ when a maximum of displacement occur that correspond to a maximum of voltage. When $SW_1$ is closed, a pseudo-periodic voltage oscillation starts around the voltage $-V_c$, until opening the $SW_1$. The switch is kept close during a half pseudo-period (the same $\Delta t_s$ for SSDI technique) of the electrical oscillator (inductance and the capacitance of the piezoelectric elements). When the switch $SW_1$ is opened the voltage has been inverted around the potential $-V_c$ and is negative. The switch $SW_2$ is controlled so as symmetric with $SW_1$. It is closed when a minimum of displacement correspond to a minimum of voltage occurs and opened after a closing time $\Delta t_s$. Consequently, the voltage sign become positive. On the other word, $SW_1$ is used during the decreasing phase of inversion, while the $SW_2$ is used during the increasing of one (i.e. the $SW_1$ and $SW_2$ switches are trigger respectively in each maximum and minimum of the piezoelectric voltage). The potential of sources are symmetric $V_c$ and $-V_c$. $V_c=0$ is correspond to SSDI technique. The temporal evolution of
the voltage and displacement associate to this technique has been represented in Fig. 2-6b.

![Diagram of electrical circuit and waves form](image)

Figure 2-6: Original SSDV technique: (a) electrical circuit, (b) waves form.

The interest of the SSDV technique is the additional term of dry friction rather than SSDI technique, which is particularly interesting out of the resonance. The SSDV technique allows increasing the efficiency of SSDI technique. The generated damping by the semi-passive technique is directly related to electromechanical coupling. For the structures with the weakly electromechanical coupling, the semi-passive technique can be ineffective. The interest of SSDV technique is that can bypass this drawback with increase of voltage on the piezoelectric elements. Applying a high voltage lead to the problem of stability, since in this case the voltage generates an excitation force instead of a brake force. Also, this technique leads to instability when the intensity of the
mechanical excitation decreases. An improvement of this technique has been proposed, that consist of inversion of piezoelectric voltage around a potential that proportional to the amplitude of the vibration. This improvement is the subject of Ref [71].

2.4.2. Modified SSDV

In the case of constant $V_c$, the instability of the system may be possible with decrease of the force amplitude. A proposed enhancement for the SSDV technique is the proportional continuous voltage to the vibration of structure (displacement) in order to eliminate the risk of instability. This can be done in the simple way that $V_c$ should be proportional and in opposite sign with the displacement. The image of strain can be monitored from the voltage on an additional piezoelectric element bonded in the same area than the patches used for the vibration control. The voltage on an additional piezoelectric element is given by Eq. (2.51), and $V_c$ by Eq. (2.52), where $\beta_v$ is the proportionality coefficient between the voltage that is in open circuit on the piezoelectric elements and voltage $V_c$ (set by the user). The electrical circuit corresponding to the modified SSDV technique has been represented in Fig. 2-7. As in the SSDI technique, the control strategy of switch consist in closing $SW$, when an extremum of displacement occurs, corresponding to an extremum of voltage $v$. $SW$ is opened after the voltage inversion. In the case of sinusoidal excitation, the expression of $V_c$ in each inversion is given by Eq. (2.53). The wave form of the modified SSDV technique is similar to the ones plotted in Fig. 2-6 (Original SSDV technique).

$$v_s = \frac{\alpha}{C_p} u$$

(2.51)

$$V_c = -\beta_v \frac{\alpha}{C_p} u$$

(2.52)

$$\begin{cases} 
V_c = -\beta_v \frac{\alpha}{C_p} u_m & \text{on a maximum displacement} \\
V_c = \beta_v \frac{\alpha}{C_p} u_m & \text{on a minimum displacement} 
\end{cases}$$

(2.53)
The work done by Badel et al. [71] has shown that the original SSDV technique causes the problem of stability with decrease of excitation force amplitude. Then, it is not more effective out of the resonance frequency. In this case, the stability problem would occur when the external driving force become lower than the force induced by the piezo-voltage. These drawbacks are efficiently by-passed by the modified SSDV technique. In the case of modified SSDV technique, the piecewise function $h$ is proportional to the amplitude of vibration. Then, it reduces rapidly when far away from the resonance. Actually, in the case of modified SSDV, $\beta_v$ can be raised to increase the damping effectiveness. But, for high values of $\beta_v$, the same stability problem than the original SSDV occurs. This raised value is $\beta_v=6$, for the work done by Badel et al. [71].
Chapter 3:

Semi-Passive Random Vibration Control Based on Statistics

It was shown that the control law consisting of triggering the inverting switch on each extremum of the voltage (or displacement) was optimal for harmonic excitation. However, in the case of complex vibrations induced by either a shock or repetitive random shocks or a random excitation force applied to a structure, the previous trivial control law does not perform well. The problem encountered is that many local maxima occur and it is difficult to determine which one should be selected for triggering the switching sequence in order to maximise the piezo-voltage and to enhance the energy extraction and damping performance on a wide frequency range of vibrations. This study considers the development of an enhanced control strategy for semi-passive piezoelectric damping devices. Following the proposed approach, the voltage or deformation signal is analyzed during a given time window and the statistically probable deformation or voltage-level threshold is determined from both the average and standard deviation of the signal during the observation period. The voltage step still
occurs on a local maximum of the signal but only above the statistically defined threshold. A significant decrease in vibration energy is demonstrated experimentally and theoretically in the case of a clamped beam excited by random noise. It is shown that in the case of multimodal vibration, all of the modes are controlled and preferentially those corresponding to the highest displacement amplitudes. This chapter presents the simulation and the experimental results under the random excitations.

3.1. Introduction

Several methods have been investigated for semi-passive nonlinear vibration damping and energy reclamation using piezoelectric elements [79–81]. These methods are interesting because they do not rely on any operating energy as in active control. They consist of drive by a few solid-state switches (i.e. MOSFET transistor) requiring very little power. The common strategy of these methods consists of modification of the electric boundary conditions of the piezoelectric elements (open or short circuit). Synchronised Switch Damping (SSD) techniques [1, 63, 81] which are implemented in this chapter consist of leaving the piezo-elements in open circuit except during a very brief period of time where the electric charge is either suppressed (SSDS) in a short circuit or inverted (SSDI) with a resonant network. The damping or energy reclamation performances depend strongly on the piezoelectric coupling coefficient and consequently this coefficient has to be maximised [82, 83]. It was shown [4] that for a harmonic regime, optimal switching should occur on each extremum of the voltage or of the piezo-element strain. Corr and Clark experimented with SSDI in the case of a multimodal vibration. The proposed technique consists of selecting the modes to be controlled using numerical filtering techniques [70]. Also, they showed that the original SSD control law was not optimal in the case of large band excitation.

In the case of large bands of excitation, the optimisation of piezoelectric elements (size and location) as well as the switching network is not sufficient. Using a nonlinear technique such as SSD is very important to operate the voltage-switching device exclusively on certain selected extrema [1, 67, 81]. As specified in the following, in the case of the SSD technique, the energy extracted from the structure by the piezo-element is proportional to \( \sum v_k^2 \), where \( v_k \) is the piezoelectric voltage just before the \( k \)th
switching sequence (short circuit or inversion). This study proposes statistical analysis to define the optimise instants for the switching sequence in order to maximise the extracted energy and vibration damping. The proposed methods are based on a statistical evaluation of the voltage generated by the piezoelectric elements or of the structure deformation. In order to theoretically analyse the various statistical methods, a beam equipped with piezo-elements wired on a SSDI switching cell was simulated and experimented.

Section 2 describes the various control techniques. Control Probability Law (CPL) is detailed as well as the proposed statistical methods. Section 3 summarises the simulation results and gives a comparison of the various methods. Finally, it depicts the experimental set up and results.

3.2. Strategies for random vibration control

When the external force is a random function, the response of the system is a random vibration. In this case, the usual SSD strategy of control which consists of triggering the inverting switch on each voltage extremum or strain extremum is not necessarily optimal. More precisely in the case of a multimodal structure such as described in Section 2.3, many extrema appear on the voltage and deflection, which corresponds, to the various modes of the structure. The switching sequence or switching strategy that allows to maximise $\sum v_k^2$ is not straightforward. This point has been illustrated in Fig. 3-1, in which it appears that the strategy of control (b) allows maximising the voltage, whereas strategy (a) allows maximising the number of inversion sequences in a given period of time.

Recent developments have been aimed at the optimal switching strategy for either vibration damping or energy harvesting. The previously investigated CPL will be briefly described and its principal drawbacks will be considered. To overcome to these drawbacks, a strategy based on statistical analysis of the voltage or deformation signal will be described. The control probability and statistics strategies are based on the idea that they allow the piezoelectric voltage to reach a significant value ($v_m$) that is statistically probable before allowing either inverting the voltage (SSDI) or forcing it to zero (SSDS). Moreover, the switch trigger will occur on the local maximum, for which
the energy stored on the piezo-element is maximum, as long as this maximum respects the previously mentioned rule.
Figure 3-1: Illustration of two different strategies for SSDI control: (a) original strategy with switching on each extremum; (b) alternative strategy consisting of selecting specific maximum (\(v\) piezo-element voltage, \(u\) structure deformation).

3.2.1. Control Probability Law

3.2.1.1. Basic principle

The CPL approach previously developed \([1, 81]\) is based on a probabilistic analysis of a signal \(x(t)\) that can be either the deformation \(u(t)\), the square of the deflection \(u^2(t)\), or the squared voltage \(v^2(t)\). It is supposed that the process can be predicted and therefore could be considered as stationary on a limited period of time (a few periods of the first mode). In this case, the cumulative distribution function \(F_{pd}(x)\) (probability distribution function) of \(x(t)\) is calculated on the estimated future time window \(T_{es}\) (Figs. 3-2 and 3-3) as defined by Eq. (3.1). From this function, the threshold value is defined according to a probability threshold \(P_{sw}\). The probability \(P_{sw}\) is established arbitrarily by the user and could depend on the performance required: vibration damping or energy harvesting.

![Probability distribution function](image)

Figure 3-2: Determination of the \(v^2_m\) threshold for the CPL method.

Figure 3-2 illustrates the probability distribution function for \(v(t)^2\). From this function, a voltage threshold \(v^2_m\) is defined according to a probability threshold \(P_{sw}\) as summarized by Eq. (3.2) and illustrated by Fig. 3-2.

\[
F_{pd}(x_m) = P[x(t) \leq x_m]
\]  

(3.1)
\[
P[v(t)^2 \geq v_m^2] = P_{sw} = 1 - F_{pd}(v_m^2)
\]

Figure 3-3: Determination of the \( u_m \) threshold for the CPL method.

3.2.1.2. Estimation of the future piezoelectric voltage or deflection

When the CPL strategy is implemented on the voltage signal, the voltage threshold has to be defined according to an estimation of the open-circuited piezo-element voltage. But this voltage has to be estimated from a piecewise continuous function before the switch instant \( t_k \) such as represented on Fig. 3-1b or Fig. 3-4. The process of voltage estimation has been illustrated in Fig. 3-4. The estimated voltage \( v_{es}(t) \) in the future time window starting at instant \( t_k \) can be theoretically obtained by Eq. (3.3) where \( t_k \) is the \( k \)th switching time at which the voltage switches from \( v_k \) to \(-\gamma v_k \). In this equation, \( q_{ik} \) is the modal coordinate \( q(t) \) of the deflection at instant \( t_k \) [81].

\[
v_{es}(t^+) = -\gamma v_k + \frac{1}{C_0} \sum_{i=1}^{N} \alpha_i (q_i(t^-) - q_{ik}) \quad \begin{cases} 
 t^+ \in [t_k, t_k + T_{es}] & \text{(future)} \\
 t^- \in [t_k - T_{es}, t_k] & \text{(past)} 
\end{cases}
\]

This is reasonable when \( T_{es} \) does not exceed a few periods of the lowest resonance frequency of the structure. The value of \( T_{es} \) is set by the user. If \( T_{es} \) is too short, the control would not be sensitive to the first mode. Conversely, if \( T_{es} \) is too long, this would induce delays in the control which would therefore decrease its frequency band. Eq. (3.3) also means that the probability distribution function and the related probability
density function of the piezoelectric voltage after the $k$th switching time are almost equal to before the switching time during the time window $T_{es}$.

![Graph showing displacement and voltage over time](image)

Figure 3-4: Estimation of voltage after an instant of switch $t_k$ for a time window $T_{es}$.

In the case of deflection, this process is more trivial because the variable is a continuous function. Therefore, the signal $x(t)$ for a period $T_{es}$ just after a switching time is supposed to be identical to the equivalent earlier window.

Experimentally, the voltage or the corresponding strain can be simply deduced from the voltage measured on an additional PZT insert left in open circuit and collocated with the principal semi-passive control PZT insert. This additional piezo-element is used as a strain sensor. If it is made with the same PZT material and with the same thickness, its output voltage $v_s$ will be the same as the voltage that would appear on the principal PZT insert left open-circuited. This voltage is given by Eq. (3.4). The subsequent future estimated voltage $v_{es}(t)$ is therefore obtained by Eq. (3.5) from the monitoring of the sensor voltage on a time window $T_{es}$ just before the $k$th switching time $t_k$. $v_k$ and $v_{sk}$ are the voltages at time $t_k$, respectively on the main PZT insert and on the collocated sensor insert.
\[ v_x = \frac{1}{C_p} \sum_{i=1}^{n} \alpha_i q_i(t) \quad (3.4) \]
\[ v_{\alpha}(t) = -\gamma v_k + v_x(t) - v_{\alpha k} \quad (3.5) \]

3.2.2. Temporal statistics method

3.2.2.1. Basic principles and definitions

The temporal statistics method [74] is based on statistical analysis of a signal \( x(t) \) that can be either the deflection \( u(t) \), the square of the deflection \( u^2(t) \), or the squared voltage \( v^2(t) \). The same assumptions as in the previous case are made concerning the reproducibility of the considered process. The statistical analysis is made on an estimation of the considered \( x(t) \) function on a time window \( T_{es} \) following any switching instant \( t_k \). The estimation of the function relies on the same principle as mentioned before. The time average or temporal average \( \mu_x \) of the \( x(t) \) signal on the \( T_{es} \) time window is defined as:

\[ \mu_x = \frac{1}{T_{es}} \int_{T_{es}} x(t) \, dt \quad (3.6) \]

The term of \( (x(t) - \mu_x) \) can be regarded as the dynamic component of a random process. In many applications, the interest lies in the mean squared value of the dynamic component. This quantity is known as the variance \( \sigma_x^2 \). Its definition is:

\[ \text{var}(x) = \sigma_x^2 = \frac{1}{T_{es}} \int_{T_{es}} (x(t) - \mu_x)^2 \, dt \quad (3.7) \]

The positive square root of the variance is known as the standard deviation \( \sigma_x \).

3.2.2.2. Statistical method on the voltage signal

This process is executed on \( v^2(t) \). The time average and standard deviation of the estimated squared voltage (Eq. (3.3) or Eq. (3.5)) are computed according to Eq (3.8).

\[ \mu_{v^2} = \frac{1}{T_{es}} \int_{T_{es}} v(t)^2 \, dt, \quad \text{var}(v^2) = \sigma_{v^2}^2 = \frac{1}{T_{es}} \int_{T_{es}} (v(t)^2 - \mu_{v^2})^2 \, dt \quad (3.8) \]

The \( v^2_{\text{th}} \) threshold will be defined by:
\[ \nu_m^2 = \mu^2 + \beta \sigma^2 \] \hspace{1cm} (3.9)

where \( \beta \) is an arbitrary tuning coefficient. The effective variations of \( \beta \) depend on the statistical distribution of the estimated \( \nu(t) \) function.

3.2.2.3. Statistical method on deflection signal

The time average \( \mu_u \) and standard deviation \( \sigma_u \) of the deflection are computed according to Eq. (3.10) on an estimation of \( u(t) \) on a \( T_{es} \) long period window.

\[ \mu_u = \frac{1}{T_{es}} \int_{T_{es}} u(x,t) dt , \quad \sigma_u^2 = \frac{1}{T_{es}} \int_{T_{es}} (u(x,t) - \mu_u)^2 dt \] \hspace{1cm} (3.10)

In this case, the positive deformation threshold \( u_m \) will be derived as:

\[ u_m = |\mu_u| + \beta \sigma_u \] \hspace{1cm} (3.11)

where \( \beta \) is still an arbitrary coefficient. According to the general principle aimed at triggering the switch once the voltage has reached a significant and statistically probable value, switching will occur when the absolute value of the deflection reaches the threshold \( u_m \).

This process can also be carried out in the case of the squared deformation. As this variable remains positive, the same definitions as for Eq. (3.9) will be used for the threshold.

3.2.3. Root mean square method

This method is based on statistical analysis of signal \( x(t) \) that is estimated on a time window \( T_{es} \) as in the previous cases. The mean squared value of a signal provides a measure of its energy. The definition of the mean squared value \( \psi_x^2 \) of a variable \( x(t) \) on \( T_{es} \) is:

\[ \psi_x^2 = \frac{1}{T_{es}} \int_{T_{es}} x(t)^2 dt \] \hspace{1cm} (3.12)

The positive square root of the mean squared valued is known as the root mean square or rms. For the various cases of \( x(t) \), the \( x_m \) threshold will be defined as:

\[ x_m = \beta \psi_x \] \hspace{1cm} (3.13)
where $\beta$ is an arbitrary coefficient. For each signal case, Table 3-1 summarises the conditions in which the instants of switch are defined.

Table 3-1: Switching condition definition for each observed signal.

<table>
<thead>
<tr>
<th>Variable $x(t)$</th>
<th>Switching condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u(t)$</td>
<td>$</td>
</tr>
<tr>
<td>$u^2(t)$</td>
<td>$u^2(t) &gt; u_m^2$</td>
</tr>
<tr>
<td>$v^2(t)$</td>
<td>$v^2(t) &gt; v_m^2$</td>
</tr>
<tr>
<td>$v(t)$</td>
<td>$</td>
</tr>
</tbody>
</table>

From Eqs.(3.6), (3.7) and (3.12), we can show that:

$$\sigma_x^2 = \psi_x^2 - \mu_x^2$$

(3.14)

Then,

$$\text{rms} \propto \sigma_x + \mu_x$$

That means, variance is equal to the mean square value minus the square of the mean value [74]. With attention to Eq. (3.14), if $\mu_x = 0$, the statistics and rms results are the same (Fig. 3-5).

![probability density function](image)

Figure 3-5: probability density function.

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3.3 Results in the case of vibration damping

3.3.1. Vibration damping performance analysis

In order to compare these different control approaches either for simulations or for experimental data, for vibration damping or energy harvesting, it is necessary to define quantities that will be evaluated for the sake of comparison. The quantity used is related to the deformation of structure. This quantity $I_u$, which is a summation in the time variable for the considered modes, is the mean squared response of $u(x,t)$ [76] and is defined as:

$$
I_u = u(x,t)^2 = \lim_{\tau \to \infty} \left( \frac{1}{\tau} \int_0^\tau u^2(x,t) \, dt \right) = \sum_i \sum_k \phi_i(x) \phi_k(x) \lim_{\tau \to \infty} \left( \frac{1}{\tau} \int_0^\tau q_i(t) q_k(t) \, dt \right)
$$

This quantity will be computed in this form for simulation results. For experimental data, it will be evaluated using either the vibrometer time response or the piezo-element sensor voltage integrated on a time window whose length is large compared to the system response. The displacement damping $A_u$ is then evaluated from these quantities, computed with the piezo-element open-circuited and then with the piezo-element driven with the desired control process, and is defined as:

$$
A_u = 10 \log \left( \frac{(I_u)_{\text{controlled}}}{(I_u)_{\text{open-circuit}}} \right)
$$

3.3.2. Numerical simulations results

In this paragraph, a random force is applied to the free end of the beam. This force is applied on a time domain lasting 200 periods of the lowest resonance frequency of the system. The model, (Eqs. (2.27) and (2.40) principally), is simulated by numerical integration using the fourth-order Runge-Kutta algorithm. The simulations are carried out using the various methods described in Section 3.2. The observation time window
$T_{eq}$ has to be twice the period of the lowest resonance frequency to give satisfactory results. This time should be sufficiently long to obtain a realistic image of the deflection especially for the lowest frequency mode and sufficiently short to maintain a good frequency response of the control. The considered structure and the corresponding numerical data calculated using the expressions detailed in chapter 2 (Section 2.3) have been gathered in Tables 3-2 and 3-3, respectively. In the following numerical simulations, the switching sequence is simply modelled at the switch triggering time by Eq. (2.3).

Table 3-2: Characteristics of the experimental structure.

<table>
<thead>
<tr>
<th>Plate material</th>
<th>Steel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plate dimensions</td>
<td>$180 \times 90 \times 2 \text{ mm}$</td>
</tr>
<tr>
<td>Piezoelectric material</td>
<td>P189 (Saint-Gobain – Quartz)</td>
</tr>
<tr>
<td>Number of piezo-elements</td>
<td>12 (on each face)</td>
</tr>
<tr>
<td>Piezo-elements position</td>
<td>10 mm from the clamped end</td>
</tr>
<tr>
<td>Piezo-element dimension</td>
<td>$10 \times 30 \times 0.3 \text{ mm}$</td>
</tr>
<tr>
<td>Piezo-elements capacitance: $C_p$</td>
<td>148 nF</td>
</tr>
<tr>
<td>Inversion coefficient: $\gamma$</td>
<td>0.6</td>
</tr>
<tr>
<td>Open-circuit resonance frequencies</td>
<td>56.48 Hz; 321 Hz; 890 Hz</td>
</tr>
<tr>
<td>Coupling coefficients: $k_i$</td>
<td>0.0959; 0.0663; 0.0265</td>
</tr>
<tr>
<td>Young’s module $E$</td>
<td>200 Gpa</td>
</tr>
</tbody>
</table>

Table 3-3: Modal parameters of the considered system.

<table>
<thead>
<tr>
<th></th>
<th>First mode</th>
<th>Second mode</th>
<th>Third mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_i$ (g)</td>
<td>71.6</td>
<td>72.2</td>
<td>71.5</td>
</tr>
<tr>
<td>$K_{di}$ (N/m)</td>
<td>7800</td>
<td>319000</td>
<td>2398800</td>
</tr>
<tr>
<td>$K_{ei}$ (N/m)</td>
<td>7700</td>
<td>317600</td>
<td>2397100</td>
</tr>
<tr>
<td>$\alpha_i$ (N/V)</td>
<td>0.0023</td>
<td>0.0102</td>
<td>0.0111</td>
</tr>
<tr>
<td>$\zeta_i$ (N/ms)</td>
<td>0.0591</td>
<td>0.3794</td>
<td>1.0357</td>
</tr>
</tbody>
</table>

Figure 3-6 shows the variations of the displacement damping $A_u$ versus both the probability $P_{xw}$ (for CPL method) and the $\beta$ coefficient (for either rms or statistics method). For the various methods, the signal considered for the definition of the threshold is $v(t)^2$. Depending on $\beta$ and $P_{xw}$, the statistics, rms or CPL methods lead to better results than simple SSDI and the damping and consequently the energy extraction are better. Note that simple SSDI consists of switching blindly on each extremum of the considered reference signal (either $u$ or $v^2$ or $u^2$). The optimum displacement damping is $–8.0 \text{ dB}$ whereas it is $–6.4 \text{ dB}$ in simple SSDI. The applicable interval of $\beta$, for which
the displacement damping $A_u$ stays minimum, is very large for the statistics method. Conversely, the other methods are more sensitive to the adjustment of either $\beta$ or $P_{sw}$. This is a good advantage for the statistics method because, for a high range of excitation signals, it is not necessary to regulate $\beta$ for each force to have optimum damping. This indicates that the target of the extremum in the case of statistics is better. The curve representative of the statistics method shows a sharp decrease in the displacement damping for $\beta$ values larger than 2. This is very understandable, because in this simulation, the quality factors of the various modes are close to $Q_m=500$ and consequently the signal is very well filtered and locally close to a sinusoid.

![Graph showing variations of displacement damping (dB) versus $\beta$ or $P_{sw}$ for different methods](image)

**Figure 3-6:** Variations of the displacement damping ($A_u$) in dB versus the probability threshold ($P_{sw}$) or the $\beta$ coefficients for the various methods. The observed signal is the squared piezoelectric voltage (Statistics, Probability, rms, Simple SSDI).

In this case, the probability density function $p(v^2)$ as shown in the Fig. 3-7 [74] is similar to sinusoidal probability density function, because in this case, the mechanical quality factor $Q_m$ is very high ($Q_m=500$), consequently the response of system to the random forces due to filtering is approximately sinusoidal.
Figure 3-7: probability density function.

Figure 3-8 shows the same kind of comparison in Fig. 3-6 but in this case the considered signal is the deflection signal \( u(x,t) \). For the statistics and rms methods, a large domain of \( \beta \) allow to have good damping. Conversely, for the CPL method, two probability thresholds have to be properly tuned on each side apart from \( P_{sw} = 0.5 \) which corresponds to the median \( (u(x,t) = 0) \). This is a drawback for the probability method when we use the deflection signal to make the switch. For the middle value \( (P_{sw} \text{ around } 0.5) \), \( u_s \) is much smaller and switching is not optimum. If \( P_{sw} \) is close to 0 or 1, this corresponds to the extremum of the deflection, which does not occur often enough to ensure good energy extraction and therefore good global deflection damping. The other methods are affected similarly when \( \beta \) is higher than \( \sqrt{2} \) which would correspond to the extrema for a pure sinusoidal signal.

Figure 3-9 illustrates this latter point. It shows the variations of the displacement damping \( (A_u) \) versus the number of switches, in the total time window. This number is naturally dependent on the control method and for the statistics method depends on the \( \beta \) coefficient. The minimum value of \( A_u \) is \(-7.0 \text{ dB} \) for 290 switching sequences for the statistics method. In the case of simple SSDI (switch on each extremum), the number of switches is 2990 for a \(-4.6 \text{ dB} \) displacement damping for the same total time window. This figure illustrates that it is not the amount of switching sequences, which is important, but much more the instant of the switch. It illustrates and quantifies the trade-off that was set by Fig. 3-1.
Figure 3-8: Variations of the displacement damping ($A_u$) in dB versus the probability threshold ($P_{sw}$) or the $\beta$ coefficients for the various methods. The observed signal is the beam tip deflection $u(L,t)$. (Statistics, Probability, rms, Simple SSDI).

Figure 3-9: Illustration of the displacement damping variations ($A_u$) versus the number of switching sequences for the statistics method for a given total time window and for different values of $\beta$. 
Figure 3-10 is similar to Figure 3-6 except that the signal considered is the square of the deflection. Another slight difference is a lower mechanical quality factor ($Q_m = 250$). In this case, the rms method tends to a lower sensitivity of the $\beta$ coefficient and the $P_{sw}$ coefficient tuning of the CPL method tends to be less sensitive.

Figure 3-10: Variations of the displacement damping ($A_u$) in dB versus the probability threshold ($P_{sw}$) or the $\beta$ coefficients for the various methods. The observed signal is the squared beam tip deflection $u(L,t)^2$. (Statistics, Probability, rms, Simple SSDI).

Figure 3.11 shows the influence of the considered signal (either $u$ or $u^2$ or $v^2$) for the statistical method. In these cases, the natural mechanical quality factor is 500. The displacement damping is plotted as a function of $\beta$. For the various signals, the optimal damping is almost the same. The main difference is related to the acceptable bandwidth for the $\beta$ adjustment parameters. It appears that using $u^2(t)$ leads to damping very little dependent on $\beta$. 
3.3.3. Experimental set up and results

The experimental set up considered is a steel beam equipped with piezoelectric inserts. Two vertical polarised piezoelectric patches have been placed in two sides of the beam (Fig. 3-12). This structure corresponds to the description given in Table 3-2. Table 3-3 data are representative of the real structure as described in Ref. [1]. The proposed control strategies are implemented using a laboratory PC-based real-time DSP controller environment (dSPACE DSP board DS-1104). Either the voltage or the deformation signals are sampled by the board as input data. Local extrema are numerically localised and the corresponding levels compared to the thresholds calculated as defined previously. According to the proposed method, the switch trigger is generated by the digital output of the control board, connected on a SSDI switching device built as described in Ref. [63]. The displacement sensor used is a simple piezoelectric insert collocated with the main insert. The sensor thickness and material
are similar to the control insert and therefore it generates the same open-circuit voltage (amplitude and phase).

Control strategy programming and implementation are done using the Matlab/Simulink™ software environment and using the dSPACE Real-Time Workshop for real-time computing and input/output control. Note that only $u(t)$ has been implemented as the observation signal for practical purposes. Since it is not necessary to estimate the signal in the future and by using a displacement sensor, it is realised very easily. It is studied during the $T_{cs}$ time window (sliding window) just before the present time and the threshold level $u_m$ is calculated as defined previously for each method, then it is compared to the observed signal to define the instant of the next switch.

Excitation of the beam is carried out using an electromagnet driven by an audio amplifier, by a random noise during a 50 s sequence generated by the dSPACE board. Actually, the noise sequence was generated once and was replayed for all of the tests for comparison. The estimation time window used is exactly twice the lowest mode period. The experimental set up has been shown in Fig. 3-13.

![Experimental sample](image)

Figure 3-12: Experimental sample.
The displacement damping $A_u$ is computed using the image of $u(t)$ given by the collocated piezoelectric deflection sensor insert. Figure 3-14 shows the experimental results for the statistics, CPL, and rms methods and simple SSDI (switch on each extremum). These results have to be compared with Fig. 3-8. Experimental and simulated data agree quite well. The weak sensitivity of the control to the $\beta$ coefficient is confirmed according to the large range of $\beta$ (approx $0.2<\beta<1.2$) which allows nearly optimal displacement damping for statistics or for rms methods.
Figure 3-14: Experimental results for the displacement damping ($A_u$) in dB versus probability ($P_{sw}$) and $\beta$ coefficients (Statistics, Probability, rms, Simple SSDI).

Figure 3-15 illustrates the beam tip velocity with and without the statistics ($\beta= 0.8$) SSDI control. The damping achieved for a large band semi-passive method (no operating energy) is remarkable. In the simulation and experiment, there is not much difference in the performance between the statistics and rms methods, because the mean value of the deflection signal is very small.
Figure 3-15: Free-end velocity of the beam: a) without control; b) in the case of SSDI control with the statistics method.

3.4. Conclusions

The SSDI semi-passive nonlinear control technique is interesting for structural damping applications because it presents simultaneously good damping performance, good robustness and very low power requirements. The main limitations are related to the case of complex or random excitation where the synchronisation on the strain extremum is not trivial.
It was shown that either a probability or statistical analysis of the strain could allow definition of criteria to identify more accurately the relevant switching instants. However, these methods require a little knowledge of the signal, which results in the definition of a tuning parameter: $P_{sw}$ for the CPL method or $\beta$ for the statistics or rms methods. The proposed work shows that for the latter methods, this adjustment can be made very coarse. Moreover, calculation of the average value or the rms value of a given signal can be easily implemented either using a numerical method as in the proposed experimental work or using analogue low power electronic circuits. It was also shown that for damping purposes the best results could be obtained using either an image of the strain or the square of this value. This is easier to implement than the reconstructed voltage signal. The main drawback in this case is the use of a specific sensor collocated with the piezoelectric insert. Global displacement damping close to 10 dB can be obtained using these approaches, nearly twice the damping that can be achieved using classic SSDI techniques. Finally, it is important to consider that these techniques are simple enough to be self-powered. In the future, the adaptation of coefficient $\beta$ and inductance with the time variations of the excitation force should be studied to attain optimum switch damping.
Chapter 4:

Semi-passive vibration suppression performance using sliding time window

In the previous chapter, it was compared the statistics method with others. In this chapter, the performance of semi-passive vibration control is investigated using this method for switching sequence to enhance the vibration damping. The proposed method for switching sequence is only based on statistical evaluation of the structural deflection. This study shows the theoretical and experimental results of this method for different excitation forces, such as stationary or nonstationary random samples and pulse forces. A significant decrease in vibration energy and also the robustness of this method is demonstrated experimentally for a clamped beam excited by different excitation samples.

4.1. Introduction

The ability to reduce the vibration amplitude over a wide frequency band is essential in vibration control. The temperature variations that can greatly influence the mechanical response of viscoelastic materials and on the other hand the change of their dynamic stiffness and damping properties with the excitation frequency cause other
methods and materials or hybrid control techniques [84, 85] to be considered. Also, there are many researches about random excitation response [86-89]. Here, it is followed by a discussion on the strategy of semi-passive vibration control by sliding time window and the proposed statistical method. This study show this new developed strategy is allowing an easy implementation of this damping technique for any type of excitation forces. The response of a cantilever beam when subjected to pulse, stationary and nonstationary random excitation has been considered. The stationary random excitation is a white noise and the nonstationary random excitations are the pulses of white noise shaped in time as either a rectangle or a half-sine.

Section 2 discusses the further strategy of semi-passive vibration control by sliding time window, Section 3 finally, summarises the simulation results and depicts the experimental set up and results.

4.2 Strategy of SSDI vibration control by sliding time window

In this case, the switching sequence is implemented based on statistical analysis of the deflection signal on a sliding time window [67]. In each instant just before the present time the temporal average $\mu_u$ and standard deviation $\sigma_u$ on a sliding time window $T_{es}$ are calculated (Eq. 3.10) and statistically probable deflection threshold is determined from Eq. (3.11). In fact, this calculation in each instant of time on sliding window is to take a decision to the next switch. This approach in the case of random vibration is very efficient. Since, in this case the most obvious characteristic is that it is nonperiodic. Knowledge of the past history of random motion is adequate to predict the probability of occurrence of various displacement magnitudes, but it is not sufficient to predict the precise magnitude at a specific instant [90]. In this method the history of signal in the past time window $T_{es}$ is studied tow times by average and standard deviation of signal. Then trigger of extremums in this case is better, especially in the case of random vibration. Also, it cause to an amplification of the amplitude of piezo-voltage.
4.3. Numerical simulation results

In this paragraph, a random force is applied to the free end of the previous clamped beam (Fig. 3-12). This force is applied on a time domain lasting 200 periods of the lowest resonance frequency of the system. The model (Eqs. (2.27) and (2.40) principally) is simulated by numerical integration using the fourth-order Runge-Kutta algorithm. The simulations are carried out using the statistics method described in Section 3.2.2. The observation time window $T_{es}$ has to be twice the period of the lowest resonance frequency to give satisfactory results.

Figure 4-1 shows the variations of the displacement damping $A_u$ versus the $\beta$ coefficient for different mechanical quality factors (different structures) under the same random forces. The applicable interval of $\beta$, for which the displacement damping $A_u$ stays minimum is very large. This is a good advantage because, for a high range of excitation signals, it is not necessary to regulate $\beta$ for each force in order to obtain optimum damping (Figs. 4-4 and 4-10 to13). This indicates that the target of the extremum in the case of statistics is better and more reliable. In the case of $Q_m=500$, The curve show a sharp decrease in the displacement damping for $\beta$ values larger than $\sqrt{2}$ which would correspond to the extrema for a pure sinusoidal signal. This is very understandable, because the signal is very well filtered and locally close to a sinusoid. The result of this filtering and pseudo sinusoidal response especially in the structures with high values of $Q_m$, is the better extremum detecting by SSDI method and resulting high damping. In the SSDI method the sinusoidal signal is the ideal signal to have an optimum switch.
Figure 4-1: Variations of the displacement damping ($A_u$) in dB versus the $\beta$ coefficients for different structures ($Q_m$). The observed signal is the beam tip deflection $u(L,t)$.

### 4.4 Experimental set up and results

The experimental set up considered is the previous steel beam equipped with piezoelectric inserts. The proposed control strategy is implemented using a laboratory PC based real-time DSP controller environment (dSPACE DSP board DS-1104) similar to before. The displacement sensor signal (as the observation signal) is studied at each instant during a $T_{os}$ time window (sliding window) just before the present time and the threshold level $u_m$ is calculated as defined previously for this method (Eq. 3.11), then it is always compared to the observation signal to define the instant of the next switch. Here, the sliding time window used is exactly twice the lowest mode period.

Excitation of the beam is carried out using an electromagnet driven by an audio amplifier, by an excitation voltage (force) generated by the dSPACE board. The controlled responses of a cantilever beam when the applied excitations are the pulse, stationary and nonstationary random forces are studied. The stationary random excitation is a Gaussian white noise and the nonstationary random excitation is a Gaussian white noise shaped in time by a rectangular envelope function or a half-sine envelope function. This appears as a pulse of white noise shaped either as a rectangle or a half-sine (random shock loadings). These vibrations with stop-and-start driving (Figs.
4-5 and 6) are clearly non-stationary; however the system experiences a series of transient vibration events. Transient vibration is nonstationary [91]. All the real random excitations are nonstationary in nature. Stationary vibration is an idealized concept. Nevertheless, certain types of random vibration may be regarded as reasonably stationary. A simple model for a nonstationary random process with a time varying mean value is given by a process where each sample record is of the form:

\[ f(t) = a(t) \times x(t) \]

Here, \( a(t) \) is a deterministic function (known physical process) and \( x(t) \) is a sample record from a stationary random process [92].

The displacement damping \( A_u \) is computed using the free-end velocity signal (the vibrometer time response) given by the laser sensor device. Fig. 4-2 shows the experimental results for this method. The weak sensitivity of the control to the \( \beta \) coefficient is confirmed according to the large range of \( \beta \) (approx \( 0.1 < \beta < 1.2 \)) which allows nearly optimal displacement damping for this method.

![Graph](image.png)

**Figure 4-2:** Experimental results for the displacement damping \( (A_u) \) in dB versus \( \beta \) coefficients. The observed signal is the displacement sensor signal.
Figure 4-3 illustrates the beam tip velocity with and without the statistics ($\beta = 1.02$) SSDI control. The damping achieved for a large band semi-passive method is remarkable; it is equal to -8.6 dB.

![Figure 4-3: Free-end velocity of the beam: a) without control; b) in the case of SSDI control with the statistics method.](image)

Figure 4-4 shows the absolute values of displacement damping for some experimental samples with approximately stationary behaviour. For the constant value
of $\beta$ (0.8) the values of displacement damping are not very different. The results show that this method of control is not very sensible to the stationary random process excitations. Because, the sliding window is always moving with signal and with the variations of signal during the time, the statistical values ($\mu$, $\sigma$) change proportionally as well. Therefore, the moments of deciding to trigger the switch are approximately optimum. It can be said, that the sliding window is independent of the type of excitation signal.

![Figure 4-4: Absolute values of displacement damping for a random process.](image)

Figures 4-5 and 6 show the white noise excitation shaped in time by either a rectangular (rectangular pulse of white noise) and half-sine envelope function (half-sine pulse of white noise), respectively. Hence, the excitation and response are nonstationary (Figs. 4-7, 8). The experimental constant (The effective time interval, $\Delta T$) of rectangular pulse of white noise is $\Delta T = 0.071$ second. It is equal to the actual time duration of the pulse and is the 4 times of natural period of the mechanical system. The half-sine envelope function is multiplied with the output of a random noise generator. This half-sine pulse of white noise has an effective time interval ($\Delta T$) of one-half the time duration of the envelope function ($T/2=0.00885$ s).
Figures 4-5 and 6 show the variations of average and standard deviation of the rectangle and half-sine pulse of random excitation responses that have been calculated on the sliding time window versus the number of sliding time window, respectively. The nonstationarity behaviour in these figures is evidence, especially on the second
moment ($\sigma$). The statistical values, $\mu$ and $\sigma$, for the controlled case are less than one in the uncontrolled case because of piezo-elements damping. The values of displacement damping on the free-end velocity of the beam for rectangle and half-sine pulse of nonstationary random excitations are -9.1688 and -8.8712 dB, respectively. In these results the value of $\beta$ coefficient is constant ($\beta=0.8$). The results show that this method of control is not very sensible to the nonstationary random process and the values of displacement damping are not many different, although the excitation forces are completely different.

Figures 4-9 show the probability density function response of the piezoelectric displacement sensor (capture signal) for the half-sine pulse of white noise sample of Fig. 4-6 with and without control on a given total time window. The probability density function describes the general distribution of the magnitude of the random process, but it gives no information on the time or frequency content of the process. Then the larger distribution indicates the greater amplitude of response. The uncontrolled signal distribution is larger than of one with control. The difference of these curves represents the value of piezo-elements damping. When the value of piezoelectric damping is higher the distribution of response signal is smaller. Also, the nonlinearity in the system characteristics influences the response probability density. In the case of random excitation, nonlinearity leads to transformation of the probability distribution making it different for the input and output processes.
Figure 4-7: Variations of time average and standard deviation of free-end velocity responses for the rectangular pulse of white noise excitation a) without control; b) in the case of SSDI control with the statistics method.
Figure 4-8: Variations of time average and standard deviation of free-end velocity responses for the half-sine pulse of white noise excitation a) without control; b) in the case of SSDI control with the statistics method.
Figure 4-9: Probability density function response of half-sine pulse of white noise excitation on a given total time window (controlled, uncontrolled).

Figures 4-10, 11 and, 12 illustrate the beam tip velocity with and without the statistics SSDI control ($\beta = 0.8$) for rectangular pulse (with effective time interval ($\Delta T$) equal to $4T$ and $100T$) and half-sine pulse of white noise samples. The values of displacement damping on the free-end velocity of beam are -8.134 dB, -7.8 dB and -8.2 dB, respectively. The damping achieved is remarkable, as well.
Figure 4-10: Free-end velocity of the beam: a) without control; b) in the case of SSDI control for the rectangle pulse of white noise excitation ($\Delta T = 4T$).
Figure 4-11: Free-end velocity of the beam: a) without control; b) in the case of SSDI control for the rectangle pulse of white noise excitation ($\Delta T=100T$).
Figure 4-12: Free-end velocity of the beam: a) without control; b) in the case of SSDI control for the half-sine pulse of white noise excitation ($\Delta T = T/2$).

Figures 4-13 show the beam tip velocity with and without the statistics SSDI control ($\beta = 0.8$) for pulse force. The pulse duration is: 0.0006 s that produced by dSPACE board. Displacement damping for free-end velocity is -5.7221 dB.
Figure 4-13: Free-end velocity of the beam: a) without control; b) in the case of SSDI control for the pulse excitation.
4.5. Conclusions

The proposed approach for the SSDI semi-passive non-linear control technique demonstrates improved performance over nearly the entire anticipated operational range. The results show that this method of control is not very sensible to all of the types of excitation behaviours (random stationary or nonstationary and pulse). Because, the sliding window is always moving with signal and with the variations of signal during the time, the statistical values change proportionally, as well. Therefore, the moments of deciding to trigger the switch are approximately optimum. It can be said, that the sliding window is independent of the type of excitation signal. Then the statistical analysis of the strain could allow definition of criteria to identify more accurately the relevant switching instants. However, these methods require a little knowledge of the signal which results in the definition of a tuning parameter $\beta$. The results show that, this adjustment can be made very coarse. This characteristic is very important in practice. In addition, Because of the filtering of excitation signal by the system, the extremum detecting (that the switch on extremums is a character of SSDI method) is become easier especially for the systems with higher values of $Q_m$. 
Chapter 5:

Damping behaviour of semi-passive vibration control using shunted piezoelectric materials

Piezoelectric transducers in conjunction with appropriate electric networks can be used as mechanical energy dissipation devices. The purpose of this chapter is the experimental observation of piezo-element damping sensitivity to variations of excitation force parameters (amplitude and frequency). In addition, the effect of the size of piezo-element area on the vibration damping in high and low values of these parameters will have been studied using SSDI technique.

5.1. Introduction

Vibration damping is one of the manifestations of the dissipation of mechanical energy related to motion in mechanical system. Sometimes just changing the system’s stiffness or mass to alter the resonance frequencies can reduce the vibration as long as the excitation frequency does not change. But in most cases, the vibrations energy needs
to be dissipated by using damping materials or devices that are tuneable with vibration. Several methods have been investigated for vibration damping, such as passive, semi-passive, semi-active, and active methods. Active control involves the use of active elements (actuators) along with sensors and controllers (analogue or digital) to produce an out-of-phase actuation to cancel the disturbance causing the vibration. All other methods that do not include a real-time active algorithm can be classified under the passive control option. Passive damping refers to energy dissipation within the structure by add-on damping devices. Viscous dampers (dashpots), viscoelastic damping [85] tuned-mass dampers [93] and shunted piezo-elements dampers are the mechanisms of passive vibration control. Some of the recent advances in using piezoceramic (PZT) systems for controlling structural vibration have been studied by Ahmadian and DeGuilio [94]. Electronic damping using piezoelectric ceramics is a less temperature sensitive and more tuneable. In this damping technique, the mechanical energy of the structure is converted to electrical energy using piezoelectric material. The electrical energy, in turn is dissipated, as heat, in an electrical shunt circuit. Most of the researches are limited to resonance frequency and they do not mention the sensitivity of piezo-elements damping to the variations of excitation force amplitude and frequency and also to the effect of the size of piezo-element area in high and low values of these parameters. In this study, the sensitivity of piezo-element damping to the variations of these parameters in the case of SSDI technique [1, 4, 63, 67, 81] is observed experimentally. And then, the effect of the size of piezo-element area on the vibration damping is studied in the high and low values of these parameters. The different behaviors of the piezo-elements are observed. The proposed method for switching sequence is based on statistical evaluation of structural deflection. It was shown that this newly developed strategy based on statistical analyses of the voltage or displacement signal allowing an easy implementation of this damping technique for any type of excitation force and also the trigger of extremum in this case is better. In order to analyse the semi-passive vibration damping methods, a beam equipped with piezo-elements wired on a SSDI switching cell was experimented.

The following section describes the energy balance equation of the system. It is followed by a discussion on the damping measurements of vibration system. The experimental set up and results are also presented.
5.2. Energy balance and energetic consideration

Vibration damping analyses are concerned with damping in terms of system response. The loss of energy from the oscillatory system results in the decay of amplitude of free vibration. In steady–state forced vibration, the loss of energy is balanced by the energy which is supplied by the excitation. This was verified by using a multimodal model of system to conclude the balance of losses energy with supplied energy (Eq. (2.46)). Energy dissipation is usually determined under conditions of cyclic oscillations. In the case of SSDI method, an electric circuit is connected to the piezoelectric elements in order to perform vibration damping or harvest energy.

Material in cyclic stress exhibits a stress-strain relation characterized by a hysteresis loop. The energy dissipated in one cycle is proportional to the area within the hysteresis loop. Consequently, the other equation of energy balance based on hysteresis loop for the system is deduced. Let us regard the following equation of motion of a vibrating system.

\[ \ddot{u} = F_i(u, \dot{u}) + F_e(t) \]  \hspace{1cm} (5.1)

Where \( F_i(u, \dot{u}) \) and \( F_e(t) \) correspond to internal and external forces, respectively.

The external and the internal loops can be then described by:

\[ F_e = F_e(u) \]
\[ F_i = F_i(u) \]  \hspace{1cm} (5.2)

We can write down the following relationship between them:

\[ F_i(u) = \ddot{u}(u) - F_e(u) \]  \hspace{1cm} (5.3)

Owing to this relationship we can convert the external loop into an internal one by subtracting the values corresponding to the external loop from acceleration dependent on displacement. Multiplication of Eq. (5.3) by \( du=\ddot{u}dt \) enables one to obtain the infinitesimal work done by exciting forces. Integrating over the entire period leads to:

\[ \int_0^r F_i[u(t)] \ddot{u}(t)dt = \int_0^r \ddot{u}(t)\dot{u}(t)dt - \int_0^r F_e[u(t)] \ddot{u}(t)dt \]  \hspace{1cm} (5.4)

The first integral on the right-hand side of Eq. (5.4) equals to zero for periodic motion. Hence, we can derive the well-known formula:

\[ L_i = L_e \]  \hspace{1cm} (5.5)
Which say that dissipated energy is equal to the work done by external force [78]. This equation that deduced from the hysteresis loop is similar to Equation (2.46) that deduced from the energy balance of the multimodal structure.

In this case, by using the SSDI strategy of control, the switching sequence is implemented based on statistical analysis of the deflection signal on a sliding time window \( T_{es} \) just before the present time such as previous chapter (Fig 5-1). Here, the deformation signal is studied during a given time window and statistically probable deformation threshold \( u_m \) is determined from both the average (\( \mu_u \)) and standard deviation (\( \sigma_u \)) of the signal during the observation period \( T_{es} \) (Eq. 3.11).

![Figure 5-1: Strategy of SSDI method.](image)

5.3 Vibration damping results

5.3.1. Damping measurements of a vibration system

There are many methods for measuring the damping of a vibration system. The level of vibration damping can be determined by dissipation coefficient (or specific damping capacity \( \psi \)) [78] that is defined as the ratio of dissipated energy within one period to the maximum energy absorbed in that period in the case of harmonic excitation.
\[ \psi' = \frac{\Delta W}{W} \]  

(5.6)

The attenuation [78] is the other parameter to compare the vibration damping with (structure with piezo-elements) and without control that defined as

\[ \text{Attenuation} = 20 \log \left( \frac{(u_M)_c}{(u_M)_{unc}} \right) \]  

(5.7)

Where \((u_M)_c\) and \((u_M)_{unc}\) are the maximum amplitude of deflection with and without control, respectively.

In practice measuring the energy dissipated per cycle, by using the concept of Eqs. (5.5) and (2.46) is a straightforward procedure [73]. For a cantilever beam to which load \(f(t)\) is applied at the free-end, this energy loss is given by the integral over a complete cycle that is defined by

\[ E_f = \int_0^T f(t)u dt \]  

(5.8)

Where, \(T\) is the cycle’s period. This integration can be estimated by using \(Z\) data points separated by \(\Delta t\), so that digital summation over \(Z-1\) data points replaces the continuous integral. Thus Eq. (5.8) becomes

\[ E_f = \frac{1}{n} \sum_{i=1}^{Z-1} f(t)u_i \Delta t = \Delta t \sum_{i=1}^{Z-1} (f_u)_i \]  

(5.9)

Where, \(n\) is the number of cycles. Therefore, by multiplying the generated force in the dSPACE board to the free-end measured velocity by laser sensor and have a dSPACE step time \(\Delta t\) for the \(Z\) captured data points during \(n\) cycles, the dissipated energy can be calculated from the Eq. (5.9), in the case of forced harmonic vibration during a time period for controlled and uncontrolled (structural damping) cases, respectively. Then the calculation of the switching damping energy (piezo-element damping), and consequently the switching damping capacity \(\psi\) from Eq. (5.6) is easy.

5.3.2. Experimental set up and results

The experimental set up considered are two steel beams of the same size equipped with piezoelectric inserts (one of them has 24 elements and other 12 elements, Fig. 5-
2). The proposed control strategy is implemented using a laboratory PC based real-time DSP controller environment (dSPACE DSP board DS-1104) and the switch trigger is generated by the digital output of the control board, connected on a SSDI switching device built similar to previous chapter. The displacement sensor used is a simple piezoelectric insert collocated with the main insert. Deflection signal is studied during the $T_{es}$ time window (sliding window) just before the present time and the threshold level $u_{th}$ is calculated as defined previously, then it is compared with the observed signal to define the instant of next switch.

Control strategy programming and implementation are done using the Matlab/Simulink™ software environment and using the dSPACE Real-Time Workshop for real-time computing and input/output control. Excitation of the beams is carried out using an electromagnet driven by an audio amplifier, by a harmonic excitation generated by the dSPACE board. The estimation time window $T_{es}$ used is exactly twice the lowest mode period. The computations have been done by using the image of $u(t)$ given by the collocated piezoelectric displacement sensor insert. The experimental results have been done in two parts, first for the variations of excitation force amplitude with constant frequency, second the force amplitude is constant and the excitation frequency changes. In this experiment the dSPACE step time $\Delta t$ is 0.00015s, that 250000 data points $Z$ is captured during 2118 period of the first mode.

![Experimental samples](image.png)

Figure 5-2: Experimental samples
5.3.2.1. Variations of force amplitude with constant frequency

Figure 5-3 show the variations of capture (displacement sensor) signal attenuation versus the force amplitude for two beams. From the figure, the attenuation decrease is limited. The minimum attenuation (ultimate value) for the beam with 24 patches is -12 dB and for the beam with 12 patches is -6.56 dB. It is observed that after ultimate value the curves rise slowly with increase of force amplitude. In this case, the structural damping is more predominant than piezo-element damping (It proportional to the square of vibration amplitude, [74, 76]). It can be concluded that the size of piezo-elements surface area is very important when the structure vibrate with low frequencies. In this figure the excitation frequency is the first resonant frequency (Table 3-2).

![Figure 5-3: Variations of the capture signal attenuation in dB versus the force amplitude.](image)

5.3.2.2. Variations of the force excitation frequency with constant amplitude

Figure 5-4 show the variations of the capture signal attenuation versus the low values of excitation frequency for the beam with 24 patches. It is observed that the attenuation is not a function of excitation frequency over a considerable frequency range. In this figure the amplitude of force is 0.94.
Figure 5-4: Variations of the capture signal attenuation in dB versus the low values of excitation frequency.

Figure 5-5 indicates the variations of capture signal attenuation versus the high values of excitation frequency for the beam with 24 patches. In comparison with Fig. 5-4, it is observed that for the high values of excitation frequency the effect of piezo-elements damping decrease (Fig. 5-8 confirms these results). For the excitation force amplitude 1.45 the attenuation approximately is -5.236 dB. Because of particular deformation of beam in high frequencies, at each instant some piezo-elements are compressed and others are stretched. In addition, the piezo-elements have been wired to each other in parallel. Therefore, some piezo-elements eliminate the damping effect of each other, and consequently the piezo-elements damping decrease (the value of attenuation in Fig. 5-4 is about twice of the value in Fig. 5-5,In spite of the fact that the force amplitude in this figure is 1.54 times more than one in Fig. 5-4). It can be said that the piezo-elements damping over a high extensive range of frequency smoothly is a function of excitation frequency (Fig. 5-8), although the structural damping is not a function of excitation frequency [74].
Figure 5-5: Variations of the capture signal attenuation in dB versus the high values of excitation frequency.

Figures 5-6 and 5-7 indicate the variations of capture signal attenuation versus the low values and high values of the excitation frequency, respectively (for the beam with 12 patches). In this case, there are not many different ones (In these figures the values of attenuation are -6.12 and -6.08 dB, respectively for the excitation amplitude 1.3). From Figs. 5-4, 5-5, 5-6 and 5-7, it is concluded that for the low range of excitation frequency the further piezo-element surface area is more efficient and for the high range the little one is more economic. With comparison between Figs. 5-7 and 5-5 for the high range of excitation frequency the difference of attenuations are more less than one in Figs. 5-4 and 5-6 for the low range of excitation frequencies. Although in Fig. 5-5, the amplitude of force is more than one in Fig. 5-7.
Figure 5-6: Variations of the capture signal attenuation in dB versus the low values of excitation frequency.

Figure 5-7: Variations of the capture signal attenuation in dB versus the high values of excitation frequency.
Figure 5-8 show the variations of $\psi$ versus the excitation frequency from low values to high values. From the figure, the value of $\psi$ is 6.85% for 100 rad/s and 4.67% for 6500 rad/s. The variations of $\psi$ between these two values is 2.18%. Therefore, the values of $\psi$ are smoothly proportional to inverse of excitation frequency. It indicates that the electromechanical converted energy decrease with increase of excitation frequency. In this figure the amplitude of force is 0.75.

Figure 5-8: Variations of switching damping capacity $\psi$ in percent versus the high extensive range of excitation frequency for the beam with 24 patches.

Figure 5-9 show the amplitude of excitation force versus the frequency. This sample force is applied during 2118 period of the first mode. In each time during this period a harmonic force is generated by dSPACE board to calculate of each point of these curves (Figs. 5-4, 5, 6, 7 and 8). It should be noted that the value of excitation frequency applied to the beam by electromagnet is half the value of response frequency. This sample harmonic force is $f(t) = 1.45 \sin (890 \pi t)$.
Figures 5-10 and 5-11 show the amplitude of capture signal with and without control versus the frequency, respectively. They are the response of the beam with 24 patches that excited with the sample force presented in Fig. 5-9. It is observed that in these figures a few resonant frequencies have been excited. The value of attenuation for these responses is -5.22 dB.

Figure 5-10: Uncontrolled response of the system.
5.4. Conclusions

Semi-passive non-linear control technique is interesting for vibration damping applications because it presents good damping performance, especially in low excitation frequencies.

The piezo-elements damping vary with respect to the size of its area and the value of excitation frequency. When the structure vibrates with low excitation frequencies, the further surface size of piezo-elements is more efficient and when it vibrates with high excitation frequencies the less surface size is more economic. Because in high frequencies in the case of more surface size, it is probable that the piezo-element damping is completely eliminated or become insignificant. This is because of particular deformation of beam (structural bending) in high frequencies. In this case at each instant some of the piezo-elements in each side of beam are in different strain sign with others in the same side (some of them are stretched and others are compressed). In addition, the piezo-elements have been wired to each other in parallel. Therefore, they eliminate the damping effect of each other. Consequently, the stored energy on the
piezo-elements (electrostatic energy) becomes little in each extremum of deflection. Then the switching operation has a little effect on damping. But in low frequencies all of the piezo-elements of each side of the beam simultaneously have the same strain sign (they are stretched or compressed simultaneously together). Then the stored energy on the piezo-elements in each extremum of deflection is maximum. In this case, when switch occur in maximum deflection, the damping is maximum. It should be noted that each row of piezo-elements along the beam width are in the same strain sign (because of beam bending condition, Fig. 5-2). Therefore, in high frequencies the less surface area of the piezo-elements operates better than the large surface area in which the piezo-elements are expanded along the length of the beam that it causes the different strain sign on the piezo-elements in the case of high frequencies. In general with increase of excitation frequency the piezo-element damping decreases. The damping capacity of switch (piezoelectric) \( \psi \) is smoothly a function of the excitation frequency. Therefore, the electromechanical converted energy decreases with increase of excitation frequency. The piezo-element damping is sensitive to the variations of force amplitude. With increase of force amplitude, the piezo-element deformation and piezo-element damping increase as well, but it is limited with respect to the size of piezo-element surface. A drawback of the model stems from the fact that the piezoelectric patch with the shunt circuit is assumed to damp vibration even if it was placed symmetrically on a vibration node, thus contradicting the basic properties of the piezoelectric patches as integral elements.
Chapter 6:

Effect of boundary (support) conditions on the semi-passive vibration control

If a piezoelectric element is attached to a structure, it is strained as the structure deforms and converts a portion of the vibration energy into electrical energy that can be dissipated through a shunt network in the form of heating. The purpose of this chapter is to present an experimental study of the semi-passive control technique sensitivity to the system boundary conditions. It is observed that the fundamental natural frequency greatly depends on these conditions. The effect of these constraints is distributed all over the system and significantly affects the results.

6.1. Introduction

In electronic damping technique using piezoelectric ceramics, the mechanical energy of the structure is converted to electrical energy using piezoelectric material. The electrical energy in turn is dissipated as heat in an electrical shunt circuit; therefore vibration energy is damped electronically. However, the boundary conditions significantly affect the results and effect of these constraints is distributed all over the system. Several methods have been investigated for vibration damping using
piezoelectric elements without any pointing to these effects. Furthermore, the damping or energy reclamation performances depend strongly on the piezoelectric coupling coefficient and consequently this coefficient has to be maximized [82, 83].

In this chapter, the effect of support conditions on piezoelectric damping in the case of SSDI method is observed experimentally. Later, the system sensitivity to these constraints is studied. The proposed method for switching sequence is based on statistical evaluation of structural deflection. In order to analyse vibration damping by this technique, a beam equipped with piezo-elements wired on an SSDI switching cell was studied.

Section 6.2 lists the various methods that can be used for the vibration damping analysis. Finally, Section 6.3 depicts the experimental set up and results.

6.2. Vibration damping performance analysis

In order to compare of vibration damping for different conditions of support, it is necessary to define quantities that are evaluated for this purpose. The first quantity is the displacement damping $A_u$ that calculated from Eq. (3.16).

The other parameter to describe vibration damping is the amplitude ratio (magnification) [78] that is define as the ratio of response amplitude (deflection) to excitation force amplitude

$$\text{amplitude ratio} = \frac{\text{amplitude of response}}{\text{amplitude of excitation force}} \quad (6.1)$$

Phase plane is the graphical representation of system energy [95]. Here, it is used for the comparison of the vibration damping in two cases with (structure with piezo-elements) and without control. Assume an autonomous conservative system that is describe by

$$\ddot{u} = f(u) \quad (6.2)$$

Where $f(u)$ is the force per unit mass. If we recall that $\ddot{u} = \dot{u} (d \dot{u}/du)$ and insert it in Eq. (6.2), yields the energy integral
\[
\frac{1}{2} \ddot{u}^2 + \nu_p(u) = E_s \tag{6.3}
\]

Where, \( \nu_p(u) = \int_{u}^{0} f(u)du \), \( \frac{1}{2} \dot{u}^2 \) and \( E_s \) are the potential energy, kinetic energy and the total energy per unit mass, respectively. Eq. (6.3) represents a family of integral curves in the phase plane, where the parameter of family is \( E_s \) (system energy). These integral curves are symmetric with respect to \( u \) axis and they are the level curves of system energy. Whenever the value of \( E_s \) becomes bigger (energy of the system goes up), the area enclosed by the curve becomes larger (This has been shown in Fig. 6-1 for a single degree of freedom system Eq. (6.4). \( M \) and \( K \) are the mass and stiffness of system, respectively) [76].

![Phase plane diagram](image)

**Figure 6-1:** Phase plane.

\[
\frac{M}{2E_m} \ddot{u}^2 + \frac{K}{2E_m} u^2 = 1 \tag{6.4}
\]

In the experimental results, capture signal (displacement sensor) is used as the image of free-end deflection of beam, (The displacement sensor is a simple piezoelectric insert collocated with the main insert. It has been placed at the clamped end of beam at which the bending moment is maximum and consequently the piezoelement deformation is maximum. Therefore, it indicates the maximum deflection in the beam, which is the free-end deflection of the beam). Then, the capture signal describes
the amount of elastic energy and the free-end velocity of beam that is measured by laser sensor describes the kinetic energy of system. Therefore in this case, the phase plane describes the amount of system energy. The smaller the close curve in the phase portrait, the less energy the system contains.

6.3. Experimental results

The experimental set-up is a cantilever steel beam equipped with piezoelectric inserts. The clamped end is tightened by six bolts (Fig. 3-12). This structure corresponds to the description given in Table 3.2. The proposed control strategy is implemented using a laboratory PC-based real-time DSP controller environment (dSPACE DSP board DS-1104). As previous chapter, local extremums are numerically localized and the corresponding levels compared to the thresholds calculated as defined previously. According to the proposed method, the switch trigger is generated by the digital output of the control board, connected on a SSDI switching device built as described in [4]. The displacement sensor used is a simple piezoelectric insert collocated with the main insert. The image of free-end deflection $u(t)$ is given by the collocated piezoelectric displacement sensor insert.

Control strategy programming and implementation is done using the Matlab/Simulink™ software environment and using dSPACE Real-Time Workshop for real-time computing and input/output control. Excitation of the beam is carried out using an electromagnet driven by an audio amplifier, by a harmonic excitation generated by the dSPACE board. The observation time window $T_{es}$ is exactly two times of the period of the lowest resonance frequency to give satisfactory results.

Figure 6-2 shows the variations of displacement damping $A_u$ in dB versus the excitation force amplitude for different conditions of clamped support. It shows that the piezo-elements damping is very sensitive to these constraints and the variations of this value for each case are different. Whenever, the tightness of support is decreased, the value of damping decreases as well. In the case of the support with clearance, the value of displacement damping is positive.
Figure 6-2: Variations of displacement damping $A_u$ in dB versus the excitation force amplitude for different support conditions.

Figures 6-3, 6-4 and 6-5 show the variations of magnification (amplitude ratio) with and without control versus the excitation force amplitude for some support conditions of the clamped-end. In these figures, the value of magnification is the ratio of capture signal amplitude (piezoelectric displacement sensor amplitude) to the excitation force amplitude. The capture signal is only the images of that part of free-end deflection of the beam that is caused by deformation and it does not show the rigid body motion. From these figures the effect of support tightness on the piezo-elements damping is evidence. Whenever the amount of support tightness becomes more, the value of strain energy and consequently, the amplitude of piezo-elements voltage increase as well. The piezo-elements voltage is only caused by deformation and rigid body motion has no effect on it. In Figs. 6-3 and 6-4 the value of magnification in the controlled case is less than the one in the uncontrolled case and their difference are increased with increase in excitation force amplitude. In these cases the piezo-elements are deformed very well, but, in Fig. 6-5 this value in controlled case is higher than the one in the uncontrolled case. This situation is caused by piezo-elements switch, which increases the amplitude.
of vibration a little. In this case (support with clearance), because of less strain energy and consequently less electrostatic energy, the piezoelectric switch can not damp much more energy in the system (Eq. 2.47) and since at the instant of switch the stiffness of the system changes from \( K_d \), open-circuit to \( K_e \), short-circuit, the elastic energy of the system will have been decreased. Therefore the kinetic energy must be increased (because of conservation) and consequently the vibration amplitude in the controlled case becomes bigger than of one in uncontrolled case. The value of magnification in Fig. 6-3 increases until 8 (in uncontrolled case) whereas, it is very low (0.6) in Fig. 6-5.

![Graph showing variations in magnification versus the excitation force amplitude for the fixed support condition of the clamped end (bolts tightened).](image)

**Figure 6-3:** Variations in magnification versus the excitation force amplitude for the fixed support condition of the clamped end (bolts tightened).
Figure 6-4: Variations in magnification versus the excitation force amplitude for zero-clearance in the support of clamped end.

Figure 6-5: Variations in magnification versus the excitation force amplitude for 0.2 mm clearance in the support of clamped end.

Figure 6-6 shows the variations of the free-end velocity amplitude of the beam versus the frequency for each support condition. The natural frequencies are extremely sensitive to the beam’s boundary conditions and support conditions. In this figure, the
values of fundamental natural frequencies are 57.49 Hz for the maximum bolt tightness; 54.18 Hz for the bolts loosened very slightly, 49.11 Hz for zero-clearance and 39.93 Hz for 0.1 mm clearance in the support of clamped end.

Figure 6-6: Variations in the free-end velocity amplitude of the beam versus the frequency for different support conditions of the clamped end.

Figures 6-7, 8, 9, 10 and 11 show the variations of the free-end velocity of the beam with and without control versus the voltage of capture signal (phase trajectories) for each support condition. From the figures, whenever the tightness of the support is decreased, the voltage amplitude of capture and consequently the value of strain energy decrease as well. Therefore, the enclosed area between two curves in the controlled and uncontrolled cases and consequently the damping energy by the piezoelectric element switch decrease. This enclosed area in the fixed support (bolts tightened) is very large (Figs 6-7 and 6-8) as compared with other conditions.

Since the switch of the SSD method occurs at the maximum deformation or maximum piezoelectric voltage, consequently maximum potential energy, the potential energy of system $E_p$ (electrostatic + elastic, that stiffness change from high-value $K_d$, open circuit to low-value $K_e$, short circuit) decrease. Because of conservation of energy, the kinetic energy increases instantaneously, resulting in the increase of velocity. In the
case of support with clearance (Figs. 6-2, 6-5 and 6-11), because of less strain energy and consequently less electrostatic energy, the piezoelectric switch can not damp much more energy in the system and since at the instant of switch the stiffness of system change, from $K_d$, open-circuit to $K_e$, short-circuit, the elastic energy of system decrease. Therefore the kinetic energy must be increased and consequently the vibration amplitude in the case of control is bigger than the one in uncontrolled case. But in the fixed condition (bolts tightened), this increase in kinetic energy in the maximum deflection is not more effective (although it is more than the one in the support with clearance), because of the high value of piezoelectric damping the vibration amplitude decreases greatly in comparison to this increase in kinetic energy.

It should be noted that in the SSD method, the system vibrates in the upper stiffness state ($K_d$) and changes to the lower state ($K_e$) once its trajectory reaches a maximum deformation. The total change in the system stiffness is small ($K_d - K_e = \alpha^2/C_p$). Therefore in the switching time, there is only a small increase in the kinetic energy, especially in the case of support with clearance. Because of the less strain energy and consequently less piezoelectric voltage amplitude, the values of $\lambda$ and $\alpha$ (Eq. (2.43)) and consequently $K_d$ (Eq. (2.41)) become small. Therefore in the support with clearance, there are not many different between controlled and uncontrolled curves. Also in this case, the electromechanical coupling coefficient $k_i$ (Eq. (2.43)) becomes small, as well.
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![Graph showing variations in free-end velocity of the beam versus capture signal voltage for zero clearance condition.]

Figure 6-10: Variations in the free-end velocity of the beam versus the capture voltage for very slight clearance in the support of clamped end with and without control.

![Graph showing variations in free-end velocity of the beam versus capture signal voltage for very slight clearance.]

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Figure 6-12 shows the variations of the capture signal and piezo-voltage in time domain for 0.2 mm clearance in the support of clamped end. In the moment of switch, because of the change in stiffness of the beam is softened, resulting very slight increase of velocity and consequently a sudden slight increase of displacement. Capture voltage in Fig. 6-11,12a and 12c show this phenomenon. On the other word, the sudden switch injects the shock in the system and consequently a sudden change in the capture signal. In Fig. 6-5, since the curves calculation is based on the ratio of maximums, the controlled curve is upper than the uncontrolled one that arises from this phenomenon. For the Fig. 6-2, in the case of support with clearance this sudden increase of capture signal makes the same problem like Fig. 6-5.
Figure 6-12: Variations of the capture signal and piezo-voltage in time domain for 0.2 mm clearance in the support of clamped end.

6.4. Conclusions

It can be concluded that the piezoelectric damping is very sensitive to the constraints of the system. The effect of these constraints is distributed over the beam’s entire length. Whenever the tightness of the support is increased, the strain energy of system increases as well. And also, the electromechanical coupling coefficient $k_i$ increases. The amount of vibration damping is proportional to amount of piezoelectric strain that this strain is affected by support conditions. The fixed condition (bolts tightened) is constrained to have higher strain energy. Consequently, the zero-clearance and then the support with clearance conditions in turn have less strain energies. In the beam with support clearance, the rigid body motion is built into the beam model. Strain is the principal factor to control vibration by piezoelectric materials. Support conditions affect the strain energy in the system and the fundamental natural frequency is highly dependent on the beam’s boundary conditions and support conditions. Finally, the boundary conditions can significantly affect the results and amount of sensitivity to these conditions was surprising. These effects are the dynamic equivalent of Saint Venant’s principle used in mechanics of materials.
Chapter 7: Conclusions

The work presented in this manuscript emphasizes the use of an intelligent material (piezoelectric materials) to develop an intelligent structure, i.e. able to self-control its level of vibration. The active electromechanical material is simply bonded on or embedded in the structure that should be controlled. It converts part of the mechanical energy into electrical energy when the structure vibrates. This energy transfer or reduction of the mechanical energy of the structure results in an effective damping of vibration. On one hand, excellent results are usually obtained using active control techniques, but these systems are large consumers of energy and also need bulky, sensitive and costly devices, including sensors for the capture of vibration, calculators to define the necessary drive signals and amplifiers and actuators for driving the structure. On the other hand, semi-passive techniques are an interesting alternative system especially when the criterions of energy, volume and cost are important. They have experienced significant development in the recent years, due to their performances and advantages compared with passive and active techniques. More precisely, SSD and
derived techniques (State Switching Absorber, SSDS, pulse switching or SSDI, SSDV, etc...), lead to a very good trade-off between simplicity, required power supply and performance. The SSDI semi-passive nonlinear control technique is interesting for vibration damping applications, because it presents simultaneously good damping performance, especially in low excitation frequencies contrary to viscoelastic materials, good robustness and very low power requirement. This technique which consists of nonlinear processing of the piezo-element voltage induces an increase in electromechanical energy conversion. In addition, it also exhibits a unique feature since the system can be self-powered directly from the converted mechanical energy without any decrease in its performance. It means that no wiring (wireless system) or power supply are needed. Moreover, the SSD techniques do not need any knowledge of the dynamical model or of the modes of the structure. It is therefore very adaptable to a multimodal structure, especially in the case of a wide-band of excitation frequency. It is less sensitive to the variations of the system parameters due to environmental or using conditions. The SSD damping techniques in comparison with active ones do not use external sources of energy; they directly pump out the vibration energy of structure in the form of electrical energy. However, it is necessary to have a little quantity of external electrical energy in order to feed the electronic control board needed to drive the transistor switches. Compared to passive techniques, performances are better either in terms of absolute damping as well as in terms of insensitivity to any tuning parameters. Semi-passive methods appear therefore as very interesting alternative techniques, rather than active or passive methods. The combination of simplicity, little mass, little volume and fair efficiency of this unique technique allows to apply it in various domains: acoustic control, vibration control or harvesting energy.

The energy extraction mechanism introduced by the SSD techniques is similar to dry friction. In fact, the nonlinear treatment of voltage creates a mechanical force in the form of piecewise function which is always out of phase with the vibration velocity. This signifies that from the view of structure the nonlinear treatment introduce a dissipative mechanism which has the same characters as dry friction.

In order to optimise the vibration damping in the case of complex or noisy vibrations, we concentrated our work on the study and the development of criteria allowing identifying accurately the relevant switching instants instead of switching
systematically on each extremum. These criteria are based on either a probabilistic or a statistical analysis of the voltage or strain signals. These new techniques are devised in such a way that the electrostatic energy on the piezoelectric electrodes always rising, except during the switching phase. However, these methods require a little knowledge of the signal, which results in the definition of a tuning parameter $\beta$. It was also shown that for damping purposes the best results could be obtained using either an image of the strain or the square of this value. This is easier to implement than the reconstructed voltage signal. The main drawback in this case is the use of a specific sensor collocated with the piezoelectric insert. Global displacement damping close to 10 dB can be obtained using these approaches, nearly twice the damping that can be achieved using the classic SSDI techniques using systematic switching. The performances of control are improved using the statistics methods, because it relies on the history of the signal in the past time window $T_{es}$. This one is accurately and simply quantified using both mean value and the standard deviation of signal. Then, the definition of the optimal triggering extremum is improved, especially in the case of random vibrations. Moreover, due to the filtering of the excitation signal by the system, the extremum detection is made easier for the structures with higher mechanical quality factor.

The proposed approach for the SSDI semi-passive nonlinear control technique demonstrates improved performance over nearly the entire anticipated operational range. The results show that this method of control is insensitive to the type of the excitation (random stationary, non stationary, pulsed …). Because, the sliding window is always moving with the signal and with the variations of the signal during the time, the statistical values adapt themselves continually. The use of an appropriate length of this sliding window is responsible for the low influence of the type of excitation on the damping performance.

Furthermore, the damping due to piezo-elements is a function of its size and according to the excitation frequency, or modes wavelength. When the structure vibrates with low excitation frequencies, the larger is the surface size of the piezo-elements, the more efficient is the subsequent damping. In the case of high excitation frequencies, there is no need in increasing the piezo-element size since positive strain regions and negatives strain ones compensate and therefore using minimal necessary surface size is more economic. In general with the increase of the excitation frequency
the piezo-element damping decreases. The damping capacity of switch (piezoelectric) $\psi$ is smoothly a function of the excitation frequency. Therefore, the electromechanical converted energy decreases with the increase of the excitation frequency. The piezo-elements damping is sensitive to the variations of excitation amplitude. With the increase of the excitation amplitude, the piezo-elements strain and piezo-elements damping increase, as well but it is limited with respect to the size of piezo-elements surface.

It was shown that the piezo-elements damping is very sensitive to the constraints of the system. The effect of these constraints is distributed over the beam’s entire length. The amount of vibration damping is proportional to the amount of piezo-elements strain. Strain is the principal factor to control of vibration by piezoelectric materials. Boundary conditions affect the strain energy in the system and the fundamental natural frequency is greatly dependent on the beam’s boundary and support conditions. Then boundary conditions could significantly affect the results and amount of sensitivity to these conditions was surprising. These effects are the dynamic equivalent of the Saint Venant’s principle used in mechanics of materials.

In order to improve the performance of this nonlinear technique, SSDV technique can be used. But, this technique leads to instability when the intensity of the mechanical excitation decreases. This drawback is efficiently by-passed by the modified SSDV technique. This can be done in the simple way that $V_c$ should be proportional and in opposite sign with displacement in order to eliminate the risk of instability. SSDV technique increases the efficiency of the SSDI technique.

The results show the good performances of the SSDI nonlinear control technique in the case of vibration damping applications. The statistical approach for defining the accurate switch instant could be extended to the field of electric energy harvesting (Synchronized Switch Harvesting (SSH)) using either piezoelectric materials for mechanical energy or pyroelectric materials for pumping on thermal energy. The modification of the conversion characteristics by the electronic devices has situated this technique in the domain of the mechatronic, and the optimisation of the switching device is corresponding to the domain of micropower.
For further improvement in the field of vibration damping, the various techniques can be used together in vibration control (hybrid control), for example the systems with the SSD techniques and viscoelastic materials. The viscoelastic materials will be used for high frequencies damping whereas the SSD technique will be focused on the low frequencies damping for which the viscoelastic materials are not effective. This association will be interesting because of its efficiency in low frequencies and the elimination by the viscoelastic absorber of harmonics generated by the nonlinear treatment in high frequency.
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[10] Wu, S., Piezoelectric Shunts with Parallel R-L Circuit of Structural Damping and


Les travaux de cette thèse concernent l'étude d'une technique particulière se rapportant au traitement de la tension générée par les éléments piezoélectriques. Cette technique non linéaire augmente considérablement l'effet de la conversion électromécanique des matériaux piezoélectriques. Cette technique appelée synchronise switch damping (SSD) a été mise au point en laboratoire de génie électrique et ferroélectricité de l'INSA-Lyon. L’un des avantages de ces techniques est la possibilité d’être autoalimenté par la conversion de l’énergie électrique par des éléments piezoélectriques. Le présent travail propose une nouvelle approche du contrôle pour les techniques SSD permettant l'amortissement dans le cas de vibrations complexes tels que les excitations aléatoires. Cette nouvelle approche est l'approche statistique sur fenêtre glissante dans le temps par rapport à la tension piézo-électrique ou le déplacement de l'ouvrage. Les résultats numériques et expérimentaux ont été présentés pour une poutre encastrée libre. Ces résultats montrent l’efficacité de cette nouvelle stratégie de contrôle, avec la capacité des patchs piézoélectrique pour amortir les vibrations de la structure. L'effet de la taille des patchs piézo-électrique sur l'amortissement des vibrations et leur sensibilité aux variations de la force d'excitation sont aussi présentées. Enfin, il montre l'effet des conditions aux limites sur la technique SSDI.

The work of this thesis concerns to study of a particular technique related to the treatment of generated voltage by the piezoelectric elements. This nonlinear technique increases the effect of electromechanical conversion of piezoelectric materials considerably. This technique called Synchronized Switching Damping (SSD) has been developed in laboratory of electrical engineering and ferroelectricity of INSA-Lyon. The advantage of these techniques is that can be self-powered by using the converted electrical energy by piezoelectric elements. The presented work propose a new approach of control for the SSD techniques that allowing to increase of damping in the case of complex vibration such as random excitations. This new approach is the statistical approach on the sliding time window of the piezoelectric voltage or displacement of structure. Numerical as well as the experimental results have been presented for a cantilever beam. These results show the ability of the piezoelectric patches with passive shunting to damp out the structural vibration and also show that this new strategy of control is very efficient. The effect of the size of piezoelectric patches on the vibration damping and their damping sensibility to the variations of the excitation force parameters have been presented as well. Finally, it shows the effect of boundary conditions on the SSDI technique.