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Multi-Objective Optimization in Short and Mid-term Home Health Care Planning

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Abstract

In France (excluding Mayotte), the number of elderly people dependent on care is projected to reach approximately 4 million by 2050. Although the total expenditure on social care has been gradually increasing, nearly 46.6% of the dependent elderly population in the European Union reported a lack of sufficient support from 2018 to 2020 (Eurostat statistics explained, 2022).

The Home Health Care (HHC) industry provides care in patients' homes, playing a crucial role in supporting the elderly, the disabled, and the chronically ill. The HHC services present a more cost-effective alternative compared to traditional hospital stays, and contribute to reducing the rehospitalization rate (Aligon, Com-Ruelle, and Lebrun, 2003; Topaz et al., 2022; Siclovan, 2018). It is mainly funded by social insurance and taxation. To meet the growing demand and save costs, it is imperative to help HHC companies make efficient planning that maximizes the utilization of limited resources while ensuring high-quality care.

In an HHC company, managers are authorized to accept a limited number of patients. The condition of patients is evaluated by their level of dependency once they are admitted. Their required services are planned on a weekly basis, with the patients being served at least once a week. Every day of the week, internal and external caregivers, who are compensated with different types of salaries and travel costs, visit patients following the routes and schedules created by managers.

For the weekly planning, our first goal is to optimally create routes and schedules considering multiple stakeholders' needs. Secondly, we aim to provide the ideal number of each type of caregiver for hire, to effectively manage the fluctuating task volumes resulting from the varying demands of admitted patients. A Mixed-Integer Linear Programming formulation is developed to address the multi-objective optimization problem. Continuity of care is emphasized given its distinguishing factor from daily care. Considering the computational complexity for large-size instances, we integrate an ϵ-greedy large neighborhood search within an improved multi-directional local search framework. We validate our model and methods using real-life data. Results show that our proposed algorithm outperforms the augmented ε-constraint method. Finally, we offer managerial recommendations to facilitate more effective employment decisions.

After establishing the weekly planning, uncertainty may arise when nurses execute these static routes and schedules daily. The uncertain service times might lead to considerable delays that adversely affect service quality. Our daily planning approach aims to create robust routes and schedules considering the uncertainty, and aid the manager in selecting a solution in practice. To this end, we introduce a new bi-objective optimization problem to model the routing and scheduling problems under uncertainty in home health care, considering the caregiver qualifications and workload. Given the computing complexity, we propose deterministic and stochastic versions of adaptive large neighborhood search embedded in an enhanced multi-directional local search framework. The results highlight the efficiency of our proposed method compared with the Gurobi Solver. The sensitivity analysis validates the robustness of the proposed model and method. Finally, we apply the method to a real-life case and provide managerial recommendations.

Résumé

En France (hors Mayotte), on prévoit que le nombre de personnes âgées dépendantes nécessitant des soins atteindra environ 4 millions d'ici 2050. Bien que les dépenses totales en matière de soins sociaux augmentent progressivement, près de 46,6% de la population âgée dépendante dans l'Union européenne a signalé un manque de soutien suffisant de 2018 à 2020 (EUROSTAT STATISTICS EXPLAINED, 2022).

L'industrie des Soins à Domicile (SAD) fournit des soins chez les patients, jouant un rôle crucial dans le soutien des personnes âgées, des personnes handicapées et des malades chroniques. Les services SAD présentent une alternative plus rentable par rapport aux séjours hospitaliers traditionnels et contribuent à réduire le taux de réhospitalisation (ALIGON, COM-RUELLE et LEBRUN, 2003; TOPAZ et al., 2022; SICLOVAN, 2018). Ils sont principalement financés par l'assurance sociale et les impôts. Pour répondre à la demande croissante et économiser des coûts, il est impératif d'aider les entreprises de SAD à planifier efficacement afin de maximiser l'utilisation des ressources limitées tout en assurant des soins de haute qualité.

Dans une entreprise de SAD, les gestionnaires sont autorisés à accepter un nombre limité de patients. La condition des patients est évaluée par leur niveau de dépendance une fois admis. Leurs services requis sont planifiés sur une base hebdomadaire, les patients étant servis au moins une fois par semaine. Chaque jour de la semaine, des soignants internes et externes, qui sont rémunérés avec différents types de salaires et de frais de déplacement, visitent les patients suivant les itinéraires et les horaires créés par les gestionnaires.

Pour la planification hebdomadaire, notre premier objectif est de créer de manière optimale des itinéraires et des horaires en tenant compte des besoins de multiples parties prenantes. Deuxièmement, nous visons à fournir le nombre idéal de chaque type de soignant à embaucher, pour gérer efficacement les volumes de tâches fluctuants résultant des demandes variables des patients admis. Une formulation de programmation linéaire en nombres entiers mixtes est développée pour aborder le problème d'optimisation multi-objectifs. La continuité des soins est soulignée étant donné qu'elle se distingue des soins quotidiens. Compte tenu de la complexité computationnelle pour les instances de grande taille, nous intégrons une recherche de

voisinage large ϵ-greedy dans un cadre amélioré de recherche locale multidirectionnelle. Nous validons notre modèle et nos méthodes à l'aide de données réelles. Les résultats montrent que notre algorithme proposé surpasse la méthode augmentée de contrainte ε. Enfin, nous offrons des recommandations managériales pour faciliter des décisions d'emploi plus efficaces.

Après l'établissement de la planification hebdomadaire, une incertitude peut survenir lorsque les infirmières exécutent ces itinéraires et horaires statiques quotidiennement. Les temps de service incertains peuvent entraîner des retards considérables qui affectent négativement la qualité du service. Notre approche de planification quotidienne vise à créer des itinéraires et des horaires robustes en tenant compte de l'incertitude, et à aider le gestionnaire à sélectionner une solution en pratique. À cette fin, nous introduisons un nouveau problème d'optimisation bi-objectif pour modéliser les problèmes d'itinéraire et d'horaire sous incertitude dans les soins à domicile, en tenant compte des qualifications et de la charge de travail des soignants. Étant donné la complexité du calcul, nous proposons des versions déterministe et stochastique de la recherche de voisinage large adaptative intégrée dans un cadre amélioré de recherche locale multidirectionnelle. Les résultats soulignent l'efficacité de notre méthode proposée par rapport au solveur Gurobi. L'analyse de sensibilité valide la robustesse du modèle et de la méthode proposés. Enfin, nous appliquons la méthode à un cas réel et fournissons des recommandations managériales.

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My heartfelt appreciation is extended to my family and my fiancé Xiaoyu for their unwavering support.

The time I have spent in France and the Netherlands has been a period of significant personal growth.

Lastly, I wish for world peace.

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To my beloved

Chapter 1

Introduction

This thesis focuses on weekly and daily planning for Home Health Care (HHC) services, considering the requirements of facility managers, caregivers and patients. This chapter introduces the background and challenges tackled in this thesis and describes the methodologies to address those challenges. The chapter is organized as follows: Section 1.1 and Section 1.2 provide the scope of HHC services and the main stakeholders. Section 1.3 describes the challenges faced in real-life cases. In Section 1.4, four research questions are addressed. The research approaches adopted in this thesis are detailed in Section 1.5. We summarize the contributions to the study field in Section 1.6. Finally, the overall structure of the thesis is presented in Section 1.7.

1.1 Context

Home Health Care (HHC) has been fast-growing worldwide in recent years with the development of information technology and transportation systems. The HHC services provide care interventions in the comfort of a patient's own home. These services are designed to support the elderly, the disabled, and the chronically ill. State-certified nurses engage in treating and monitoring the symptoms presented by patients, while other caregivers participate in maintaining patients' quality of life by contributing to their hygiene, mobilization, movement, and comfort. The mission of these services is to help patients receive treatment in their familiar surroundings and avoid hospitalization or shorten hospital stays (Rodriguez et al., 2015). The demand for HHC services has been steadily increasing in recent years due to the rapid growth of the elderly population and the rising incidence of chronic diseases. 46.6% of the EU elderly population (aged 65 or above) with difficulty in personal care or household activities stated that they did not receive adequate assistance from 2018 to 2020 (Eurostat statistics explained, 2022). As of January 2020, 13.7 million people residing in France are 65 years old or older, making up 20% of the population. This proportion has increased by four points in 20 years. France (excluding Mayotte) would have 4 million elderly people in a situation of dependency by 2050. In 2014, France spent 30 billion euros (1.4% of GDP) on elderly care, with public funds covering 23.7 billion euros (1.11% of GDP), and projections suggest that by 2060, public spending could rise to 2.07% of GDP, with total spending almost doubling to 2.78% of GDP (Bajeux, Corvol, and Somme, 2021). To meet this growing demand, innovative and efficient care delivery models that make the most of limited resources while ensuring high-quality care for their patients are needed.

The integration and coordination of this health service logistic network is a complex task and managers have to face many logistics decisions. Network design, transportation management, staff management, and inventory management are all described as the set of decision problems related to design and operation in HHC (Grieco, Utley, and Crowe, 2021). Depending on the time horizon from long term to short term, the planning horizon can be categorized into three levels: the strategic level, the tactical level, and the operational level (Gutiérrez, Gutiérrez, and Vidal, 2013). The strategic level decisions involve defining facility locations, patient districts, transportation modes, staffing, and service levels. The fleet assignment to patients' districts, the shift scheduling, and the definition of inventory policies are considered at the tactical level. The decisions at the operational level include staff assignment and routing, as well as inventory control.

1.2 Stakeholders in home health care services

1.2.1 Authorities and providers

In the governance framework in France, the departmental council (Le Conseil Départemental) confers official authorization to HHC service providers for the provision of support to the elderly with loss of autonomy as well as to individuals with disabilities. The service providers have the status of medico-social service, a designation that obligates them to comply with rigorous regulatory standards and uphold the rights of their users. The department plays an active role in monitoring the quality of services provided by these organizations.

Home care is financed through a mix of social insurance premiums and general and local taxation. The providers receive public and private funds, in the form of a flat rate paid for daily care given to each patient (Genet et al., 2013). These payments vary depending on how intensive the care is, the patient's level of dependence, and the length of stay. The allocation and oversight of this funding are managed by the Regional Health Agencies (Agences Régionales de Santé, ARS). Expenses related to the provided care are covered through a comprehensive care allowance. In 2014, the total cost of elderly care in France was estimated at 30 billion euros. This cost is made up of 41% health expenses, 35% specific care for loss of autonomy (including 52% for APA - Allocation Personnalisée d'Autonomie), and 24% for accommodation expenses in facilities (Isabelle et al., 2022). There is an incentive to optimize costs while considering service quality.

Concerning healthcare services, the HHC providers offer an array of specialized treatments, including home nursing care services Services (Soins Infirmiers à Domicile, SSIAD), and home-based hospitalization (Hospitalisation à Domicile, HAD). The services of SSIAD facilitate home living for individuals post-hospitalization or to avoid institutional care, and are fully funded by health insurance funds (Assurance Maladie). The HAD provides continuous and coordinated medical and paramedical care at the patient's home, distinguished by the complexity and frequency of treatments from other home care services. The services range from general medicine to oncology, with palliative care and complex wound dressings as common treatments. The costs of HAD are covered by health insurance and mutual funds under the same conditions as a standard hospitalization, except for the hospital fee.

1.2.2 Caregivers

The staff employed by HHC providers includes registered nurses, nursing aides, coordinators, and administrative personnel. The nurses and caregivers involved in the SSIAD provided comprehensive care, including medical monitoring, personal hygiene assistance, and various paramedical services, with continuous availability even on Sundays and holidays. Specialized roles like occupational therapists or psychologists may also be part of the team, depending on the organization's size and focus. Employment models vary: some HHC providers employ salaried nurses (internal caregivers), while others collaborate with independent nurses through contractual agreements (external caregivers). They are assigned to visit a group of patients at home. They travel to patients' homes using various modes of transportation, such as public transit, and private vehicles. Workload balance and satisfaction are essential for caregivers to prevent burnout, ensuring consistent and effective care for recipients.

1.2.3 Beneficiaries

The determination of the necessity for home care services in France is made by a medical professional. This responsibility falls either to the patient's primary care physician or to a hospital physician in cases where the patient has undergone hospitalization. The assessment for elderly or disabled individuals' care needs is typically done at home by approved professionals. The process doesn't always require a GP's referral but can include it, especially in cases of pre-existing diseases. After the assessment, the General Council develops a care plan and commissions decide on the services to be provided based on this plan. In France, eligibility for the SSIAD is based on the need for medical supervision, absence of hospitalization requirement, availability of local home care, and being over 60 or ill/dependent, with possible age reduction for early aging or disability upon medical approval. Elderly individuals experience a certain degree of loss of autonomy. For patients, they require services that last a certain amount of time within a time frame. Caregivers must adhere to patients' fixed availability time frames; violating these "hard" time windows is prohibited, while "soft" time windows can be breached with a penalty. Punctuality and continuous care are crucial as they foster trust, prevent health complications, and optimize long-term outcomes.

1.3 Home health care weekly and daily activities

The weekly activities are shown in Figure 1.1. Patients register with a HHC company after receiving a prescription. Managers collect their information and required services to make planning. Patients' required services are repeated on a weekly basis, with the patients being served at least once a week. Managers are authorized to accept a limited number of patients in a certain geographical area and decide who to serve and who to place on the waiting list (Gomes and Ramos, 2019). This limitation creates workload volatility due to two main factors: the changing severity of health conditions among patients on the waiting list, and management's ambition to expand the range of services offered. The workload may exceed or fall below the maximum work capacity of the existing two types of staff, i.e., internal caregivers and external caregivers. The all-external-staffing model could meet the demand, but at the expense of losing the internal competence of the role (DeVaro, 2020). It might be harder to maintain consistent quality and control over services. More internal caregivers create the risk of stranded or under-utilized resources. Caregivers within the HHC provider, both internal and external, start from different depots and receive different forms of payment. The payment structure includes care remuneration for external caregivers, as well as fixed salaries and overtime compensation for internal ones. Currently, the company lacks a quantitative framework to guide task allocation and long-term staffing decisions.

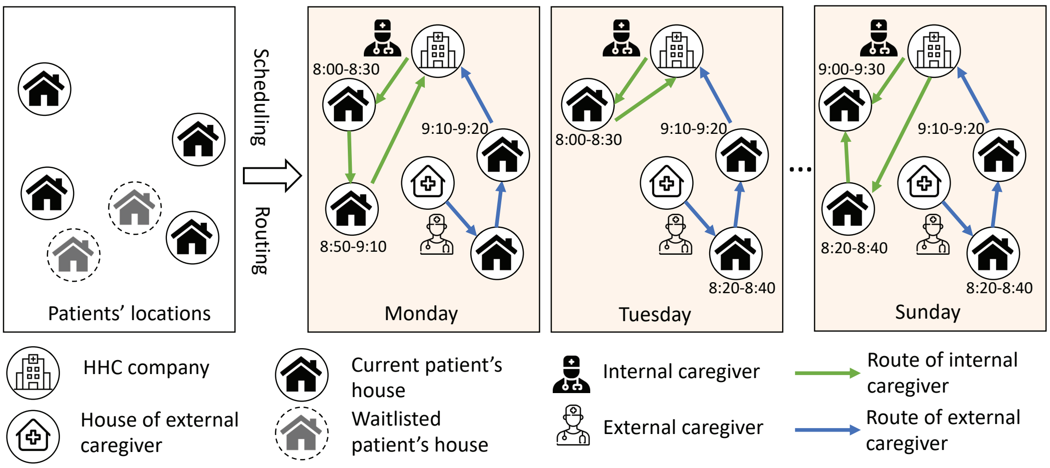


FIGURE 1.1 – Weekly activities in Home Health Care.

Each day in a week, after a set of geographically dispersed patients receive their medical prescriptions and service details, managers allocate caregivers and set schedules based on service needs and patient availability. Each caregiver delivers their medical service, starting from the home health care facility, and visiting patients in a planned sequence before returning to the facility (Yang, Ni, and Yang, 2021).

After establishing a weekly plan, uncertainty may arise when nurses execute these static routes or schedules daily. The time that a caregiver spends on the road changes little, except for instances of major road accidents. The uncertainty mainly stems from uncertain service times caused by factors such as varying patient conditions, especially in emergencies or acute cases, the challenges of addressing social isolation in elderly patients, and the diverse individual needs for social interaction. Some patients might require additional time and attention for meaningful conversations or activities than the scheduled service times. Caregivers may arrive earlier or later than expected at the next patient's home, impacting service quality, patient satisfaction, and inadvertently extending work hours. For instance, timely insulin injections are essential to maintain blood glucose levels (Fikar and Hirsch, 2017). Similarly, time-regulated nutritional support is needed in palliative care (Holdoway, 2022). Therefore, establishing robust routes and schedules for real-life home health care activities is crucial.

1.4 Research questions

This thesis aims to propose optimization models and methodologies for HHC service planning, taking into consideration the satisfaction of both caregivers and patients, with the ultimate aim of enhancing cost-effective HHC service operations. In light of the challenges outlined in Section 1.3 and the research gaps identified in Chapter 2, the main research question of this thesis is listed as follows.

— **Q1:** How the optimal weekly routes and schedules can be created while accommodating the needs of three different stakeholders?

The caregivers prioritize a balanced workload and seek to minimize wait times at patients' homes. The patients underscore the importance of timely service and continuous caregiver visits. The manager in an HHC company aims to control expenses, focusing on reducing travel and staff-related costs. To meet the specific requirements of all the stakeholders, the multi-objective optimization model and method are crucial to obtain a Pareto front, which highlights optimal trade-offs between different goals.

— **Q2:** What is the ideal number of each type of caregiver to hire, to effectively manage fluctuating task volumes?

By determining the number of caregivers, HHC providers can make informed decisions about hiring and allocation, leading to potential cost savings and improved service quality. Most studies assume uniform salaries for all staff, our study differentiates between internal and external caregivers. Our proposed model operates under the assumption of an unlimited number of caregivers. We optimized the model to determine the ideal number of each type of caregiver. This optimization is particularly relevant when considering different patient compositions and varying workload scenarios.

— **Q3:** How can the robust daily routes and schedules be established while considering uncertain service times, as well as the satisfaction of both caregivers and patients?

In the daily HHC planning, the ability to respond to uncertain service time has emerged as a vital capability to ensure the quality of care. However, few studies have balanced operational costs with stakeholder satisfaction under such uncertainties. In our study, we present a bi-objective optimization model, incorporating stakeholder needs and uncertain service times. The Pareto front was obtained by utilizing a novel stochastic method for the problem under uncertainties.

— **Q4:** After we solve the problem and obtain solutions, how can we aid the managers in selecting a solution from the Pareto front in practice?

After deriving solutions (the routes and schedules), making an appropriate selection is crucial to ensure efficacy in meeting organizational goals. Managers need to choose a solution that aligns with their business objectives. We apply our proposed method to a real-life case, provide detailed characteristics of several solutions on the Pareto front and give useful management recommendations.

1.5 Research approach

The approach of this research is presented in Figure 1.2. To address research questions Q1 and Q2, Chapter 3 develops the mathematical model and solution algorithm for the weekly planning with heterogeneous staff. To address research questions Q3 and Q4, Chapter 4 formulates a daily planning model considering uncertainty and develops a stochastic heuristic method.

Firstly, a weekly planning including routes and schedules for caregivers is needed for HHC activities, given their recurrent nature on a weekly basis. To address this,

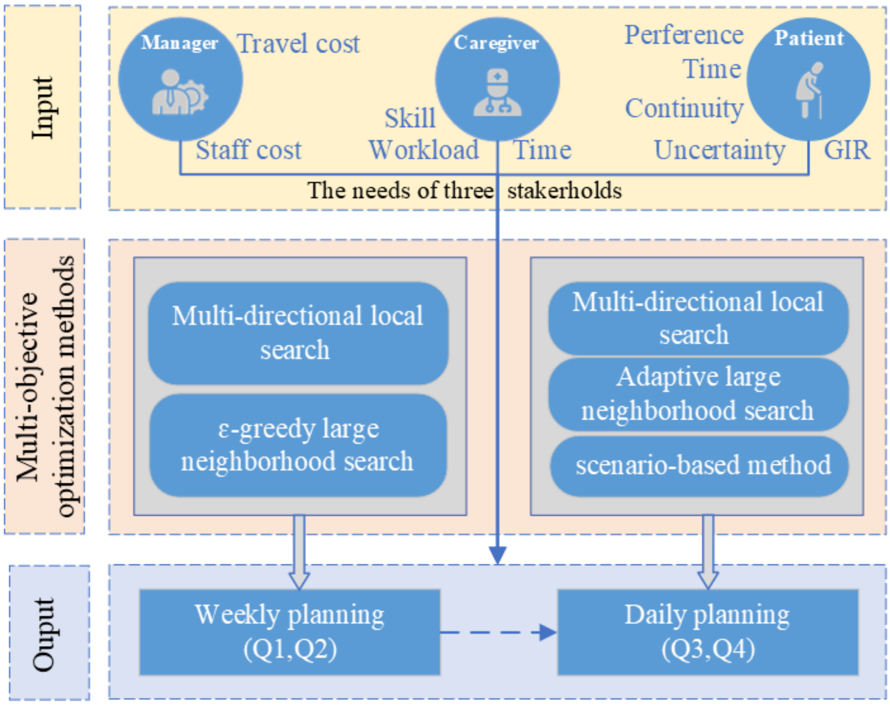


FIGURE 1.2 – Research approach.

we built a multi-objective optimization model tailored to the diverse needs of stakeholders. To solve this mathematical model, we compare the augmented ε-constraint method and an ϵ-greedy Large Neighborhood Search (ϵ-gLNS) method within the Multi-Directional Local Search (MDLS) framework. Through the MDLS, we generate solutions for each objective by heuristic methods and establish the Pareto front using the non-dominated check. The ϵ-gLNS within the MDLS, which is a heuristic method, iteratively optimizes the initial solution. Based on the mathematical model, algorithm, and real-world data, we achieved the weekly planning that satisfies various stakeholders and determined the optimal number of different types of caregivers for different scenarios.

Secondly, research on daily planning for HHC activities planning is conducted to address service time uncertainty and provide more robust planning. The bi-objective model seeks to balance cost savings with the satisfaction of both patients and caregivers, taking into account specific needs. While the Gurobi Solver can only handle small instances, we develop an Adaptive Large Neighborhood Search within the MDLS framework (ALNS-MDLS) for the problem with larger sizes. The stochastic method is used to deal with uncertain service times, which integrates a scenario-based method with the ALNS-MDLS. The proposed model and method under uncertainty are applied to a real-life case, yielding the Pareto front. We examine the features of some solutions and offer managerial recommendations in choosing routes and schedules.

1.6 Thesis contributions

This thesis contributes to home health care logistics by developing a series of approaches to solve routing and scheduling problems. The main contributions are summarized from two perspectives: one being the weekly routing and scheduling problem, and the other the daily routing and scheduling problem.

The main contributions of the weekly routing and scheduling problem are as follows.

— We construct a novel multi-objective mixed-integer linear programming formulation for the weekly HHC planning to satisfy three stakeholders in a French HHC provider (case study). It is the first to determine the number of internal and external staff.

— We propose a novel method named ϵ-greedy Large Neighborhood Search embedded in an enhanced Multi-Directional Local Search to solve the multi-objective optimization problem.

— We derive several managerial recommendations from a real case study in France that contribute to a better understanding of the problem and guide employment decision-making processes.

The key contributions of the study on the uncertain daily routing and scheduling problem can be summarized as follows:

— We develop a new bi-objective mixed-integer linear programming model that highlights the relationship between travel costs and the satisfaction of both caregivers and patients. It also incorporates the alignment of caregivers' qualifications with patients' requirements as well as workload balance.

— We propose the ALNS-MDLS to solve the problem. The effectiveness of the new approach is validated by experimental results thanks to the comparison with the Gurobi solver. The stochastic ALNS-MDLS is proposed to deal with uncertainties. The contrast between the stochastic and original versions demonstrates the stochastic method's robustness.

— In order to refine and enhance the application of our method, we conduct a sensitivity analysis to identify suitable parameters, and apply them to real-world data in a case study, providing actionable management recommendations to choose the suitable schedules.

1.7 Thesis outline

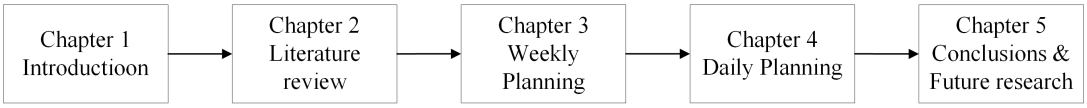


FIGURE 1.3 – Thesis structure.

Figure 1.3 shows the outline of the thesis. A literature review is conducted in Chapter 2. In Chapter 3, a mathematical model is developed for the weekly planning with heterogeneous staff. To solve the medium- or large-sized problem instances of the proposed multi-objective model, a heuristic algorithm with customized operators is proposed. In Chapter 4, building upon the research problem in Chapter 3, the daily routing and scheduling problems under uncertain service times are introduced and a stochastic method is proposed to address this problem. Chapter 5 concludes the thesis and provides directions for future research.

Chapter 2

Literature review

First, this chapter presents a comprehensive review of the logistic problem in the Home Health Care (HHC). Second, we focus on the routing and scheduling problem in this field, exploring related aspects that include the planning horizon, diverse objectives and constraints, solution methodologies, and the inherent uncertainties. By comparing various studies, we have identified research gaps which will help address the research questions we proposed in Chapter 1.

2.1 State-of-art problems in home health care operations

Planning in manufacturing focuses on efficiently allocating resources to meet customers' needs while coordinating resources and product flows to achieve organizational goals (Hans, Van Houdenhoven, and Hulshof, 2011). In healthcare, a patient is a customer that receives care and treatments. The resources refer to the input to deliver health care services, e.g., operating rooms, medical equipment, and staff. Providers aim to create a more efficient service delivery system that satisfies both patients and medical staff (Brandeau, Sainfort, and Pierskalla, 2004). Hulshof et al. (2012) categorized various healthcare services, including ambulatory care, emergency care, surgical care, inpatient care, home health care, and residential care services. Home health care has emerged as a cost-effective alternative that boosts the efficiency of healthcare providers. The visits at home by nurses facilitate medication reconciliation, clinical assessments, and continuity of care. Aligon, Com-Ruelle, and Lebrun (2003) demonstrated that costs in the HHC context are less expensive than in traditional hospitalization. Recent research has indicated the early initiation of home care services post-hospitalization can significantly reduce rehospitalization rates and costs for patients (Topaz et al., 2022; Siclovan, 2018; Li, 2024).

Home health care services operate on a delivery network, and managers have to face many logistics decisions, including network design, transportation management, staff management, and inventory management (Grieco, Utley, and Crowe, 2021). Depending on the time horizon from long term to short term, the human resource planning in the HHC can be grouped into three levels: the strategic, tactical, and operational levels (Gutiérrez, Gutiérrez, and Vidal, 2013).

At the strategic level, the organization's mission is translated into long-term decisions, involving defining fleet sizing, staffing levels, facility locations, and patient districts. The location-routing model was built and solved to save costs for home care providers (Dai, Zhang, and Chen, 2023; Fard, Hajiaghaei-Keshteli, and Paydar, 2018). They designed the suitable location of the HHC center and made the route plans. Staff dimensioning is vital for HHC structures as slight changes can significantly impact their economic performance and sustainability. The number of staff in HHC was determined to attend demand, considering skill combinations, demand fluctuations, and location uncertainties (Rodriguez et al., 2015). Tactical planning focuses on organizing the execution of the healthcare delivery process. The fleet assignment to patients' districts is mainly considered at this level. The districts were created by grouping multiple territorial basic units to achieve an optimized multi-criteria objective. Each district is managed by a specific team of nurses. Blais, Lapierre, and Laporte (2003) designed the districts that were easy to travel by public transit. Benzarti, Sahin, and Dallery (2013) assigned the teams to the districts while considering the workload balance and the compactness. The operational level is related to the short-term decisions tailored to individual patients and resources, e.g., the staff assignment and routing. The task assignment and the staff scheduling problems were addressed by Moosavi, Ozturk, and Patrick (2022). They assigned the medical tasks to permanent and part-time staff members on each shift during the planning horizon, emphasizing cohorting policies and limiting staff exposure. Turner, Mehrotra, and Daskin (2010) involved employing temporary or float staff during specific shifts to address sudden patient surges or staff shortages. Existing research often addresses routing and scheduling problems, where the manager determines not only the routes that the caregivers should take (routing) but also when they should reach each patient's home (scheduling). (Cissé et al., 2017).

There is a significant interrelation between hierarchical levels. Naderi et al. (2023)

integrated decisions of three levels. They solved staffing, assignment, routing, and staff scheduling problems. Restrepo, Rousseau, and Vallée (2020) integrated staff dimensioning and staff scheduling, considering demand uncertainty. Their proposed model first addressed caregiver staffing in districts, then tackled reallocation, contacting staff on days off, and demand adjustments. These decisions were influenced by longer-term staffing plans. Higher-level decisions dictate the scope of lower-level ones, while feedback from healthcare delivery informs higher-level decision-making (Hulshof et al., 2012).

In this thesis, we have formulated route plans and schedules at the operational level, which can be modeled as a Home Health Care Routing and Scheduling Problem (HHCRSP). Additionally, we determined the number of staff, which impacts long-term employment decisions at the strategic level.

2.2 Home health care routing and scheduling problems

The Home Health Care Routing and Scheduling Problems (HHCRSP) are an extension of the Vehicle Routing Problem with Time Windows (VRPTW). Objectives, constraints, solving methods, and different planning horizons are summarized in this subsection.

The HHCRSP is also an NP-hard problem because determining the optimal solution to VRPTW is NP-hard (Desaulniers, Desrosiers, and Spoorendonk, 2011). When solving large-scale instances, exact algorithmic methods sometimes fail to reach the optimal solution. Heuristic and metaheuristic methods are also widely employed to solve the problem (Cissé et al., 2017).

2.2.1 Vehicle Routing Problem with Time Windows

The Vehicle Routing Problem (VRP) is a well-researched combinatorial optimization problem found in numerous real-world applications, e.g., food delivery (Zhang and Chen, 2014), logistics distribution (Konstantakopoulos, Gayialis, and Kechagias, 2022), and waste collection (Han and Ponce Cueto, 2015). The VRPTW extends VRP by requiring customer service to begin within a time window which is the customer's available time frame. For a hard time window, performing the service is not permitted if a vehicle arrives or leaves outside of the time window. A soft time window can be violated at the cost of a penalty. The VRPTW is defined on the directed graph $G = (V, A)$, where $V = \{0, 1, 2..., n + 1\}$ is the vertex set. The depot is denoted by vertices 0 and $n + 1$, which are known as the source and sink vertices, respectively. The customer vertex set is represented by $N = V \backslash \{0, n + 1\}$. Every feasible vehicle route matches the source-to-sink paths in the graph G. However, the opposite might not always hold true because of the time window constraints. A traversal distance (time) and a time window are denoted as $t_{i,j}$ and $[a_i, b_i]$, respectively. The service time s_i is given. Assuming that the travel distance (time) matrix satisfies the triangle inequality, feasible solutions exist only if

$$a_0 \leq \max_{i \in V \backslash \{0\}} \{b_i - t_{0i}\}, \tag{2.1}$$

and

$$b_0 \geq \max_{i \in V \backslash \{0\}} \{\max\{a_0 + t_{0i}, a_i\} + s_i + t_{i,n+1}\}. \tag{2.2}$$

Note that an arc (i, j) in A can be excluded due to time constraints, if $a_i + s_i + t_{ij} > b_j$. The objective function can be minimizing the number of vehicles utilized. In this case, the arc $(0, n + 1)$ with $t_{0,n+1}$ must be included in A.

2.2.2 Objectives and constraints

The HHCRSP consists of designing optimal routes to deliver these services over the planning horizon, considering minimizing cost and maximizing service quality. It is subject to certain operational constraints with variations stemming from differing properties and regulations across countries (Grenouilleau et al., 2019). It differs from the VRPTW because of the features (Fikar and Hirsch, 2017): (1) the temporal dependency and the disjunctive nature of services; (2) the continuity, given that patients are assigned to a restricted set of caregivers during the planning horizon; and (3) caregivers' skills and patients' requests. We grouped these features based on the stakeholders in the HHC, namely managers, caregivers, and patients.

Managers

Managers in HHC companies strive to minimize operational costs, which primarily consist of staff salaries and travel costs, to enhance the overall efficiency and sustainability of the care system. Minimizing the travel cost is common in the literature as transportation services constitute a substantial portion of a company's expenses. Environmental issues are garnering increased attention. Recent studies have shed some light on reducing the CO_2 emission (Kordi, Divsalar, and Emami, 2023). Besides cost reduction, the manager also seeks to achieve a more balanced solution. The study conducted by Carello, Lanzarone, and Mattia (2018) showed a limited increase in expenditures achieved good quality of service. The results obtained by Braekers et al. (2016) indicated that if costs were increased by a specific fraction of their range, inconvenience could be significantly reduced. The planning horizon and frequency of scheduling updates are also determined by managers (Cissé et al., 2017).

Caregivers

Caregivers can be categorized as full-time (salaried) or part-time (non-salaried), and they may possess various skill sets. Salaried nurses receive pay for a full-time shift daily, regardless of their actual scheduled hours. They earn overtime pay when working beyond the regular shift duration. Meanwhile, part-time nurses have costs proportional to their total working hours (Cheng and Rich, 1998). Cheng and Rich (1998) minimized a single objective function which was the staff cost associated with overtime and scheduled part-time work. They did not consider the travel cost differences between the two kinds of care workers. Slaugh and Scheller-Wolf (2023) found that assigning part-time nurses higher priority than full-time ones minimizes inconsistency in long-term care facilities. The daily total working time can not be violated according to the local legal provisions of varying countries. For instance, the daily limit in the UK is 7.5 hours (Akjiratikarl, Yenradee, and Drake, 2007), while the weekly limit is 48 hours in France. Another important labor regulation being considered in the literature is the break. Nurses can travel to fictitious nodes within break time windows (Nasir and Dang, 2018). Incorporating lunch/temporary breaks into the mathematical model is challenging. One method involved creating a break node for each caregiver, ensuring its feasibility within the lunchtime window Trautsamwieser and Hirsch (2014). The other approach introduced idle time between two patient visits for breaks (Bard, Shao, and Wang, 2013). Caregivers must have qualifications/skills that align with patient needs, making certain nurse-job pairings infeasible. The match between skills and needs was predefined as a binary variable by Xiao, Dridi, and El Hassani (2018). Ciré and Hooker (2012) assumed the skill levels were parameters. Simpler jobs can be assigned to more highly skilled caregivers, while more complex jobs cannot be completed by less skilled ones. In order to maintain optimal care quality and prevent burnout, it is crucial for caregivers to avoid being overloaded with responsibilities and to achieve a balanced workload. The workload can be assessed based on the number of patients and working hours. Workload balance can be determined by comparing the workload between pairs of caregivers (Liu et al., 2021b). This balance can also serve as a constraint to limit the total workload allocated to each caregiver (Carello, Lanzarone, and Mattia, 2018). Durak and Mutlu (2024) considered workload caused by ergonomic risks and proposed a workload assessment method. Nurses move to the patients by various transportation modes, e.g., bicycles, private cars, or public transportation. Fikar and Hirsch (2015) identified the optional routes that nurses walked to the subsequent clients. Four types of vehicles with different speeds and emissions were considered by Kordi, Divsalar, and Emami (2023). Milburn and Spicer (2013) provided an option that some patients received visits from remote monitoring devices, which would not incur travel costs. Caregivers using various vehicles or with different contracts begin and end their work at different locations, leading to a multi-depot problem (Liu et al., 2021b; Trautsamwieser, Gronalt, and Hirsch, 2011).

Patients

The quality of care from patients' perspective is influenced by several aspects, which are difficult to separate and capture with a mathematical function. Most of them are also related to the nurses' working conditions, encompassing the viewpoints of both service providers and nurses. The service level was measured by assessing the suitability of a care plan based on patient-caregiver preferences and the extent to which patients' preferred time slots were satisfied (Duque et al., 2015). Zhang et al. (2021) maximized the satisfaction rate with respect to waiting time and lacking inter-operation time from the patient's perspective. Decerle et al. (2018) evaluated the quality of care by delivering care during a patient's availability and, for synchronized visits, ensuring both caregivers were present simultaneously. Care services are provided within time windows, which is a feature for maintaining quality of care. Xiao, Dridi, and El Hassani (2018) and Aguiar, Ramos, and Gomes (2023) established constraints to guarantee that services were scheduled strictly within hard time windows. A piecewise function calculated the penalty for deviations from the soft time window. The function's value varied based on which zone the arrival time fell into (Decerle et al., 2018). The continuity of care can be divided into time consistency and continuity of the caregivers. The empirical study of Ma et al. (2021) showed higher continuity reduced rehospitalization risks. When incorporating new patients into the master schedule, some patients could only tolerate minor deviations from the original plan (Gomes and Ramos, 2019). Tellez et al. (2022) considered service time consistency, aiming to serve patients at similar hours during the planning horizon. By categorizing similar times into classes, they improved consistency by reducing the overall time classes for all patients. Patients were categorized into three groups based on care continuity needs: those requiring a single nurse throughout the entire planning horizon, those needing a dedicated nurse per period, and those without specific restrictions (Carello, Lanzarone, and Mattia, 2018). The consistency was measured as the total number of nurses visiting the patients by Milburn and Spicer (2013). Güven-Koçak et al. (2024) defined consistency as the unchanged ratio of patient–aide assignments between consecutive scheduling periods, ensuring it through a rolling horizon approach. Patients may have preferences for specific caregivers and may require treatments, such as insulin injections, at particular times. The patients expressed their preference for specific characteristics of caregivers (Malagodi, Lanzarone, and Matta, 2021). Rejections for personal reasons, such as a client's incompatibility with a nurse, were predefined by Trautsamwieser, Gronalt, and Hirsch (2011). In the study of Nasir and Dang (2018), a patient had three states, i.e., newly admitted, referred to other services, or assigned to the waiting list. They minimized the patients' dissatisfaction which was associated with patient referral and placement on the waiting list. Akbari et al. (2023) defined the urgency and severity of each patient's condition using a triage level. They minimized the total weighted waiting time, with the weights corresponding to these triage levels, in order to prioritize urgent patients and increase the quality of care. Some patients require multiple visits in a single day, with some visits having clinical precedence relationships, while others necessitate simultaneous collaborative care (Hashemi Doulabi, Pesant, and Rousseau, 2020; Bazirha, Kadrani, and Benmansour, 2023; Aguiar, Ramos, and Gomes, 2023; Rasmussen et al., 2012).

In most other studies, the time window is used to limit the service starting time. The service starting time outside the time window leads to a penalty due to patients' dissatisfaction. However, it is more reasonable that the time window is defined as the time frame during which the patient is available. There will also be a penalty if caregivers perform a service and then leave the patients' homes outside the time windows. We, therefore, define patients' and caregivers' satisfaction as minimizing the segmented penalty due to arrival times and departure times being out of the time windows. We define the penalty as a piecewise function. The penalty value is assigned based on the time exceeding the time window falling into different intervals. If we simply use the total time exceeding the time window as the penalty value, it could lead to a situation where only a few patients bear the brunt of significant delay. Existing literature seldom addresses the distinct travel and staff costs simultaneously. Few studies evaluate workload based on a patient's condition, recognizing that worse patient health implies increased task complexity for caregivers. In our weekly routing and scheduling problems, the internal and external caregivers are paid differently. We consider the fairness of workload consisting of both working time and task complexity. In the daily planning, we assume a fixed number of caregivers to be assigned in the daily schedule. The number of patients to be served by one caregiver is limited for the sake of balancing the workload. We first consider these properties together.

2.2.3 Solving methods

We have reviewed the modeling techniques and solving methods in this subsection. The HHCRSP can be modeled using mathematical programming approaches, e.g., Integer Linear Programming (ILP) (Akbari et al., 2023) and Mixed-Integer Linear Programming (MILP) (Kordi, Divsalar, and Emami, 2023). Stochastic Programming (SP) is utilized in scenarios involving uncertainties (Bazirha, Kadrani, and Benmansour, 2023). Constraint Programming (CP) is applied to get the initial solution (Hiermann et al., 2015). Because the problem is NP-hard, it costs a lot of time to get optimal solutions when testing larger-size instances by commercial solvers. Given the computational challenge, alternative solution methods such as heuristics, metaheuristics, and matheuristics are employed.

Exact methods

These methods have the advantage of achieving optimal solutions. However, it usually takes a long time to execute. A branch-and-price algorithm was developed by Rasmussen et al. (2012). Using Dantzig-Wolfe decomposition, they modeled the home care crew scheduling problem as a set partitioning problem with side constraints and solved it via dynamic column generation in a branch-and-price framework. It involved creating feasible staff schedules in a subproblem and choosing these in the master problem to maximize visit coverage. The branching phase addresses the generalized precedence constraints and the integrality constraints that were relaxed in the master problem. Trautsamwieser and Hirsch (2014) used a branch-price-and-cut method to address a mid-term routing and scheduling problem, breaking it down into a master problem for weekly planning and subproblems for daily tours. They employed a labeling algorithm for the subproblems and refined the master plan with new tours derived from dual variables. Branching was used when no columns and cuts can be added to the master problem and the current solution has variables that are fractions.

Approximate methods

Heuristics are tailor-made procedures that apply problem-specific knowledge to guide the search process, while metaheuristics are higher-level frameworks that adaptively modify the heuristic strategies to explore the solution space more broadly. Grenouilleau et al. (2019) proposed a constructive heuristic to build an integer solution for the resolution of a set partitioning model. The savings algorithm is a kind of constructive heuristic and it is often used to construct the initial solution (Akjiratikarl, Yenradee, and Drake, 2007). Simulated Annealing (SA) and Tabu Search were applied in two phrases by Shahnejat-Bushehri et al. (2019). Local search operators, e.g., 2-opt, 2-opt*, and relocated move, were used to generate new neighborhoods for these two metaheuristic approaches. (Liu, Tao, and Xie, 2019) proposed an Adaptive Large Neighborhood Search approach with some tailored removal and insertion operators. The results of testing Benchmark instances showed the ALNS outperformed the commercial solver and the SA. Population-based algorithms, e.g., a particle swarm optimization (Akjiratikarl, Yenradee, and Drake, 2007) and Genetic Algorithm (Liu et al., 2013; Bazirha, Kadrani, and Benmansour, 2023) were used to tackle the HHCRSP.

Matheuristics

This method indicates a blend of mathematical programming techniques with heuristic algorithms. They are particularly useful for complex problems where traditional exact algorithms might be computationally infeasible, and pure heuristics may not guarantee a sufficiently good solution quality. Matheuristics often involve breaking down the problem into smaller, more manageable parts that can be solved using exact methods, and then using heuristic techniques to piece together these parts into a global solution. Allaoua et al. (2013) decomposed the problem into a set partitioning-like problem and a Multi-Depot Traveling Salesman Problem with Time Windows (TSPTW). Heuristic methods were used to get feasible solutions for the TSPTW. Liu et al. (2021b) combined the Adaptive Large Neighborhood Search (ALNS) with the solver. A simplified mathematical model was solved by the commercial MIP solver. The ALNS and the solver were used interchangeably to update the solution.

2.2.4 Planning horizon

Most researchers focus on daily planning problems. The decision-makers make routes and schedules for a single working day. To our knowledge, operational research was first used in the HHC planning problem by Begur, Miller, and Weaver (1997). They determined the daily travel routes and schedules for nurses to visit. By integrating geographic information system software, scheduling heuristics, and databases, a spatial decision support system was developed into a user-friendly tool, which achieved lower costs and improved the workload balance among nurses. Tanoumand and Ünlüyurt (2021) presented a mathematical formulation for the resource-constrained home health care vehicle routing problem and solved it by a branch-and-price algorithm. Liu et al. (2021a) developed four hybrid metaheuristics aimed at generating high-quality plans for HHC companies while adhering to a range of real-life constraints.

When the planning horizon extends beyond a single day, research in the field tends to focus on either mid-term or long-term planning. Patients have a set visit frequency during a designated period, and receiving care from familiar caregivers is preferred. Thus continuity of care is an essential indicator to evaluate the care quality. In the literature, some studies propose limiting the number of caregivers assigned to patients during the planning horizon (Liu et al., 2021b), while others suggest providing continuity of care for a week and rotating caregivers among patients to prevent conflicts and musculoskeletal injuries (Gomes and Ramos, 2019). According to the results presented by Güven-Koçak et al. (2022), updating the scheduling while maintaining consistency led to an increase in cost, highlighting the significant trade-off that existed between cost and continuity of care.

In the context of mid-term planning, the typical planning period is a single week, requiring managers to devise a plan for the entire week at once. The goal of Liu, Xie, and Garaix (2013) was to minimize the maximum weekly routing costs associated with transporting drugs between the HHC depot and patients' homes using various Tabu search and local search schemes. Patients received one or more visits per week from skilled caregivers (Trautsamwieser and Hirsch, 2014). The maximum working time per week, rest times between consecutive working days as well as weekly rest times were planned. Compared to mid-term planning, long-term planning focuses on even more extended time frames, such as months or even years. The manager needs to decide the frequency to update the schedules. Changes in care demands are often considered. The problem can be resolved when patients are dropped from the schedule or new patients are added from the waiting list (Gomes and Ramos, 2019). Bennett and Erera (2011) presented a rolling horizon myopic planning approach with a new capacity-based insertion heuristic to schedule visits for patients over a prescribed number of weeks, while considering the remaining available time in the nurses' schedules. Cire and Diamant (2022) considered uncertainty associated with future demand and the practitioner's tour. A Markov decision process was used to model the decision of assigning patients to health practitioners.

In our study, we consider a weekly HHC problem in Chapter 3. The services are recurrent on a weekly basis for the weekly planning. For example, a patient receives nursing care three times a week (prescribed by doctors), undergoing a 30-minute session on Monday morning, a 20-minute session on Wednesday morning, and another 30-minute session on Friday afternoon. Although the start times and duration of care vary each day within a week, these schedules are consistent on a weekly basis.

2.2.5 Multi-objective optimization

In multi-objective optimization (MOO), there is seldom a single optimal solution due to multiple objectives (Marler and Arora, 2004). Pareto optimality identifies solutions where no other feasible solution exists that improves one objective without worsening others. A feasible solution x has no other feasible solution x' such that $f_i(x') \geq f_i(x)$ for all i, with at least one strict inequality. The Pareto front is the graphical representation of Pareto optimal solutions in the objective space. Since generating the entire Pareto front is often computationally challenging and we cannot prove that our proposed algorithms can find the entire Pareto front for our problem, for simplicity, the term "Pareto front" in the following text will also encompass the approximate Pareto front, similarly for the Pareto optimal solutions (for more concepts see (Arora, 2017)).

A single solution method can be used for solving MOO if the decision-maker's preference is known before the solution process. One of the most intuitive methods is to optimize the weighted sum of all the objective functions as a single objective optimization. It is implemented simply but is highly dependent on weights. The bounded objective function method minimizes the single most important objective function, while others are used to form additional constraints. The optimization process of the lexicographic method is done individually on each objective function following the order of importance and stops when a unique solution is obtained. In a game theoretic approach, objective functions are assumed to be the players which are ultimately controlled by the decision-maker and can be expected to reach an agreement, meaning the game is cooperative. This was proposed in (Rao, 1987) and an improvement was stated in (Ghotbi, 2013). However, it is hard for the manager to know the importance of each objective beforehand. It is better for them to make decisions from a set of solutions when they can't quantify their preferences explicitly. In the ϵ-constraint method, one selected objective function is optimized while others are used as constraints. Multiple solutions are obtained by changing the right-hand side of the constrained objectives. Normal-boundary intersection is a technique intended to find the portion that contains the Pareto optimal solutions, delineating the boundary of the set of attainable objective vectors (Das and Dennis, 1998). Evolutionary algorithms simultaneously process a set of possible solutions, known as a population. Multiple solutions are updated in a single run based on information from previous iterations and random factors. The Non-dominated sorting genetic algorithm, the strength Pareto evolutionary algorithm, and their extended versions are popularly used to generate Pareto optimal front (Coello, 2006). Recently, reinforcement learning has been applied to multi-objective problems as well. The Pareto Q-learner algorithm has been developed to learn deterministic, non-stationary, non-dominated multi-objective policies, and to identify the entire Pareto front (Van Moffaert and Nowé, 2014; Leng et al., 2022). Besides, if the managers do not specify any preference or tend to seek their preferred solutions iteratively, no-preference methods and interactive methods can be used (Hwang and Masud, 2012; Miettinen, Ruiz, and Wierzbicki, 2008).

Most articles in the HHCRSP domain focus on solving a single objective optimization problem. Only transportation cost was minimized by Tanoumand and Ünlüyurt (2021). Decerle et al. (2018) combined distance and visit penalties into a single objective function, finding genetic algorithms and local searches yielded instance-flexible results. Grenouilleau et al. (2019) minimized the weighted sum of travel time, a score of continuity of care, overtime, idle time, and penalty for unscheduled patients. The objective functions related to operational costs and quality of care were integrated into one objective function with weighted coefficients by Malagodi, Lanzarone, and Matta (2021) and Intrevado, Verter, and Tremblay (2019). Recent progress has moved to explore the MOO to obtain Pareto optimal solutions instead of relying on a weighted objective function. Yang, Ni, and Yang (2021) optimized the operating cost, caregivers inconsistency and workload balance by using an improved multi-objective artificial bee colony metaheuristic. The trade-off between operating cost and service level was addressed by Braekers et al. (2016). Fathollahi-Fard, Hajiaghaei-Keshteli, and Tavakkoli-Moghaddam (2018) considered travel costs and CO_2 emissions as objective functions, using hybrid versions of metaheuristics and developing four fast heuristics for Pareto optimal solutions.

Although improving service quality and patient satisfaction is as important as reducing costs, there is still less related research on the subject. This has motivated us to introduce a multi-objective model to reconcile the interests of different stakeholders in HHC. We do not depend on the preference of decision-makers (for example, weights assigned to the objectives) to aggregate the objectives into one. Decision-makers can select their preferred solution from a Pareto optimal set according to different operational situations. This approach is easy to implement and ignores the gradient information and the nature of objective functions and constraints. It contains fewer hyperparameters compared to genetic algorithms.

2.2.6 Uncertainty

Parameters represented stochastically can capture uncertainties such as service times, travel times, and patients' demands. Two common models used in general formulations are the Chance Constrained Programming (CCP) model and the Stochastic Programming with Recourse (SPR). Li, Tian, and Leung (2010) conducted a comparative study of these two models for the vehicle routing problem under uncertain travel times and service times. Based on the results obtained by Tabu Search, the authors concluded the CCP might not be a suitable model for the target problem since the computational difficulty arose from strong constraints imposed by the confidence levels. Consequently, the stochastic programming model has been chosen for handling the uncertain service times in our study.

The robust optimization aims to obtain robust solutions that remain relatively unchanged under uncertainties. The uncertainties can be modeled deterministically, probabilistically, or possibilistically (Beyer and Sendhoff, 2007). The uncertainties in HHC can be mainly quantified based on certain distributions (Yang, Ni, and Yang, 2021; Shi et al., 2018; Bazirha, Kadrani, and Benmansour, 2023), the triangular fuzzy numbers (Fathollahi-Fard et al., 2020; Bahri, Talbi, and Amodeo, 2021), the budget uncertainty polytopes (Shi, Boudouh, and Grunder, 2019), and a set of scenarios with fixed probabilities (Naji, Masmoudi, and Mellouli, 2017; Shiri, Ahmadizar, and Mahmoudzadeh, 2021). The worst-case philosophy and expected performance of all scenarios can be employed to construct the robust objective function of the original formulation. The former, though conservative, can yield impractical solutions when overly large domains are chosen. Therefore, we define the robust counterpart of the original objective function based on the expectation. We assume the uncertainty in our study follows a normal distribution. If solutions are obtained with optimal expected values while involving uncertainties that are random variables or follow probability distributions, it can be also called stochastic optimization.

To solve the HHCRSP under uncertainties, some studies in the HHCRSP utilize numerical techniques to transform the robust optimization problem into a normal optimization problem by using strong mathematical assumptions. Yuan, Liu, and Jiang (2015) used an approximate formula to replace the expected penalty for late arrival and thus reduce computing effort. The objective functions and constraints related to uncertain service time and travel time were considered by Yang, Ni, and Yang (2021) and transformed into their deterministic equivalents based on uncertain theory. This consistent HHCRSP was solved by an improved multi-objective Artificial Bee Colony (ABC) metaheuristic. The objective function was rewritten as a recursive function based on the theory of budget in (Shi, Boudouh, and Grunder, 2019). However, this approach may be limited by the need for strong mathematical assumptions like first- or second-order derivatives, which may not always be accessible. In most studies, the robust objective function is typically calculated by the simulation method, and then exact, matheuristics, or metaheuristic methods are applied (Shi et al., 2018; Bazirha, Kadrani, and Benmansour, 2023). The scenario-based method is often employed to simulate expected function values through corresponding sample average approximation. It generates a finite set of scenarios, each symbolizing a potential realization of the uncertain parameters (Ghilas, Demir, and Van Woensel, 2016).

In other fields, some researchers apply scenario-based methods to solve a stochastic multi-objective model (Tosarkani, Amin, and Zolfagharinia, 2020; Gao and Cao, 2020; Liu, Qiao, and Kong, 2019). A multi-objective optimization problem under uncertainty in transmission expansion planning was proposed by (Maghouli et al., 2010). The objective functions were the total cost, the robustness and the flexibility criterion. The proposed process for solving this problem considered the performance of solutions in all scenarios simultaneously. It can be applied to a situation where there are not too many scenarios because the model needs to be optimized under each scenario. In the micro-grid operation field, Niknam, Azizipanah-Abarghooee, and Narimani (2012) modeled load demand, available output power and market price, by means of scenario-based stochastic programming.

In Chapter 4, to obtain daily robust routes and schedules, we have developed the stochastic model and method. Given a certain distribution of the service time, we aim to optimize the expectation of the objective functions instead of using only the mean of the service times for a deterministic model. It is hard to calculate the integration of complex objective functions. Sampling from the distribution into a finite set also generates a big increase in the number of variables and constraints. Therefore, we introduce a stochastic method that combines a scenario-based method with our proposed metaheuristic method to deal with uncertain service times.

2.3 Conclusions

The literature review is summarized in Table 2.1. These papers, sourced from authoritative journals, closely align with our research theme of routing and scheduling in home health care. Some recently published papers have been included to represent the latest developments in the field. They cover the key research focuses within this domain, featuring various planning durations, diverse models and methodologies, and differing objectives and constraints. This broadens our research perspective and lays a theoretical foundation for the feasibility of our study. We conclude some main research gaps by comparing our study with the literature that is closely related.

— The existing research mainly has centered around single-objective optimization. A single solution fails to accommodate different stakeholders' needs and operational situations, leading to reduced comprehensiveness and flexibility.

— There is a lack of research that considers both internal and external caregivers, as well as the evaluation of caregiver numbers in response to fluctuating patient demands.

— There are no similar studies in the field of home health care that have explored the scenario-based stochastic method to address multi-objective optimization problems under uncertainty.

Notably, Pahlevani et al. (2022) focused solely on minimizing travel cost, and Nasir and Dang (2018) considered a weighted sum of objectives such as driving time, overtime, and violations of time windows, with both studies yielding a single solution. In contrast, our study has addressed the multi-objective optimization problem. We have obtained Pareto optimal solutions, offering decision-makers the flexibility to choose a solution that best aligns with their specific priorities. Decerle et al. (2018), Gomes and Ramos (2019), and Kordi, Divsalar, and Emami (2023) optimized three objective functions to save costs, as well as satisfy caregivers and patients. However, we are the first to minimize both travel and staffing costs across two types of caregivers. Pahlevani et al. (2022) and Yuan, Liu, and Jiang (2015) incorporated minimizing caregiver numbers directly into their objective functions. Our study innovates further by deciding the number of two types of caregivers in response to fluctuating patient demands. We have classified patients by considering the varying levels of autonomy loss, recognizing that each level corresponds to distinct care demands. Yuan, Liu, and Jiang (2015) and Bazirha, Kadrani, and Benmansour (2023) modeled the HHCRSP under uncertainties by stochastic programming formulation. Yuan, Liu, and Jiang (2015) modified the objective function and constraints with stochastic service times by an approximate formula, while Bazirha, Kadrani, and Benmansour (2023) used Monte Carlo simulation. However, they only focused on single-objective optimization problems using Monte Carlo simulation or the scenario-based method. Our study is the first to apply the scenario-based stochastic method in addressing the multi-objective optimization problem in this domain. The multi-objective optimization model and algorithm we propose offer a broader range of choices where manager preferences are unknown.

In this chapter, we present a general literature review of HHCRSP, focusing on the feasibility of research questions and identifying gaps in current studies. Firstly, we introduce common problems across strategic, tactical, and operational levels. Secondly, we review a diverse range of research on the HHCRSP. Most studies expand upon the VRPTW model with specific objectives and constraints, vary in planning horizons, and sometimes include uncertainty. Compared with existing models, we identify areas where research is currently lacking: re-evaluating the satisfaction of patients and caregivers; differentiating compensation among various types of caregivers; addressing the optimization of their respective required quantities; and incorporating considerations for patients with varying health conditions. Thirdly, we summarize the solving methods, including both exact and approximate approaches. This can assist in choosing suitable methods for research and in understanding the strengths and limitations of different approaches. We have chosen the multi-directional local search method to address our multi-objective optimization problem. The local search algorithm can be improved and customized. The scenario-based method presents a potential approach for managing uncertainty, yet its application in multi-objective optimization problems within this field remains unexplored. Therefore, we propose novel models and algorithms to address our problem. In this study, we aim to model the weekly and daily HHC routing and scheduling problem as a multi-objective optimization problem, effectively addressing the needs of three key stakeholders: managers, caregivers, and patients. After generating a stable weekly schedule, we adjust the daily schedule considering uncertain service times. A stochastic method is proposed to address uncertainty. Based on the analysis of the results, we will offer managerial recommendations. Decisions on the number of caregivers will provide a reference for long-term employment planning. Managers can then choose a solution based on their preferences.

Authors	Objectives	Constraints	Model	Solving methods	Uncertainties	Horizon
Cheng and Rich (1998)	SOO; overtime for salaried staff and cost for non-salaried staff	Time windows compatibility and feasibility, lunch break, limited shift length	MIP	Heuristic	Deterministic	Daily
Akjiratikarl, Yenradee, and Drake (2007)	SOO; travel distance, # unassigned activities	Time windows, total daily working hours	-	Particle swarm optimization	Deterministic	Daily
Kordi, Divsalar, and Emami (2023)	MOO; travel cost, emission, workload imbalance and service quality	Time windows, staff service levels, patient preference, different vehicle types	MILP	Multi-objective variable neighborhood search	Deterministic	Daily
Nasir and Dang (2018)	SOO; travel cost, hiring cost, staff dissatisfaction cost (idle time and workload difference), patients dissatisfaction cost (referral and sending to waiting list)	Mandatory break, time windows, total daily working hours, qualification compatibility, contract duration	MILP	Variable neighborhood search	Deterministic	Daily
Liu et al. (2021b)	SOO; working duration, # reassignments of caregivers for each patient, workload imbalance	Time windows, lunch break, caregiver qualifications, synchronized visits, total daily working hours, continuity of care	MIP	Matheuristic	Deterministic	Multi-period
Liu et al. (2021a)	SOO; wages of caregivers, costs of vehicles	Time windows, caregiver qualifications, lunch break, synchronized visits, flexible departure modes of caregivers	MILP	Hybrid genetic general variable neighborhood search	Deterministic	Daily
Carello, Lanzarone, and Mattia (2018)	MOO; overtime costs, workload imbalance, # reassignment	Time windows, total working hours per period	MILP	Multi-criteria optimization, CPLEX solver	Uncertain treatment duration, robust cardinality-constrained formulation	long-term, rolling horizon
Decerle et al. (2018)	MOO; working time, time windows non-satisfaction, synchronized visits non-respect, maximal working time difference (workload imbalance)	Time windows, caregiver qualifications	MILP	Memetic algorithm for multi-objective optimization	Deterministic	Daily

TABLE 2.1 – A summary of the literature in the HHCRSP Part 1

Trautsamwieser, Gronalt, and Hirsch (2011)	SOO; driving times, waiting times, overtime, unfilled preferences, # time window violations, # overqualified nurses, unpaid driving times	Caregiver qualifications, working time restrictions, time windows, mandatory break	MILP	Variable neighborhood search	Deterministic	Daily
Bazirha, Kadrani, and Benmansour (2023)	SOO; travel costs, patients' delayed of services operations and caregivers' extra working times	Time windows, caregiver qualifications, staring service time synchronization	SPR	Genetic algorithm, variable neighborhood search	uncertain travel and service time, Monte Carlo simulation	Daily
Akbari et al. (2023)	SOO; waiting time of patients with weights corresponding the triage levels	Only basic constraints to ensure feasible routes	IP	Generalized variable neighborhood search	Deterministic	Daily
Rasmussen et al. (2012)	SOO; travel cost, caregiver preferences, uncovered visits	Time windows, temporal dependencies modelled by generalised precedence constraints	MILP	Branch-and-price	Deterministic	Daily
Gomes and Ramos (2019)	MOO; travel time, changing visit times for current patients, workload imbalance	Time windows, lunch break, daily and weekly loyalty, non-loyalty between weeks, minimum # visits to be selected from the waiting list, deviation for starting times	MILP	Matheuristic	Deterministic	periodic, rolling horizon
Yang, Ni, and Yang (2021)	MOO; travel cost, service cost, late arrival penalty, caregiver inconsistency, workload imbalance	Total daily working hours, time windows	CCP	Multi-objective artificial bee colony	Uncertain travel and service times defined by uncertain theory	Multi-period
Cire and Diamant (2022)	Wage cost associated with the working time, # rejected referrals	Time windows, patient requirements, continuity of care, shift length, workload balance	MDP	Approximate dynamic programming approach	Uncertain demand	Multi-period
Braekers et al. (2016)	MOO; travel cost, overtime cost, client inconvenience measured by deviation from preferred starting time and dislike of nurses	Time windows, total daily working hours, caregiver qualifications	MIP	Large neighborhood search in the multi-directional local search	Deterministic	Daily

TABLE 2.1 – A summary of the literature in the HHCRSP Part 2

Trautsamwieser and Hirsch (2014)	SOO; total working time	Time windows, total daily working hours, daily break and weekly break, caregiver skills	MILP	Branch-Price-and-Cut	Deterministic	Mid-term
Pahlevani et al. (2022)	SOO; travel distance	Time windows, # available caregivers, patient requirements and preferences, expected range of working time	MILP	Ordering Points and Agglomerative Hierarchical Clustering	Deterministic	Daily
Shi, Boudouh, and Grunder (2019)	SOO; travel cost, fixed costs of caregivers	Time windows, caregiver skills, # patients visited by a caregiver	MIP	Gurobi Solver, Simulated Annealing, Tabu Search, Variable Neighborhood Search	Uncertain travel and service times defined by theory of budget uncertainty	Daily
Fathollahi-Fard et al. (2020)	MOO; facility allocation cost, routing cost, transportation cost, ↑ patients' satisfaction (the privilege adopted from patients to caregivers)	Allocation of pharmacy and laboratory, vehicle capacity, time windows	MILP	Adaptive memory strategy	Uncertain travel parameters and patients' satisfaction, fuzzy approach	Multi-period
Shiri, Ahmadizar, and Mahmoudzadeh (2021)	MOO; transportation costs, overtime, locating cost, overqualified skill, ↑ score of location selection	Open facilities, time windows, route duration, maximum # nursing teams assigned to one team, caregiver skills, maximum overtime	MIP	Nimbus method	Uncertain service time, scenarios with fixed probabilities	Daily
Aguiar, Ramos, and Gomes (2023)	SOO; travel and service times	Continuity of care, lunch break, synchronized visits, time windows, maximum route length, maximum # caregiver teams	MIP	Commercial solvers	Deterministic	Daily
Yuan, Liu, and Jiang (2015)	SOO; travel cost, # caregivers, expected service cost and penalty for late arrival	# patients visited by a caregiver, caregiver skills	SPR	Branch-and-price	Uncertain service times, an approximate formula	Daily
Zhang et al. (2023)	SOO; average waiting time	Time windows, acquaintanceship, total daily working hours, caregiver skills, patient preferences	CCP, MDP	Reinforcement learning, ant colony optimization	Uncertain service times, deterministic equivalents	Daily

TABLE 2.1 – A summary of the literature in the HHCRSP Part 3

Chapter 3	MOO; travel costs for two types of caregivers, fixed salary and overtime pay for internal caregivers, compensation for external caregivers, penalty for exceeding time windows, workload imbalance	Time windows, the maximum daily working time, continuity of care	MILP	ϵ-greedy large neighborhood search in a multi-directional local search framework	Deterministic	Mid-term
Chapter 4	MOO; travel cost, penalty for exceeding time windows	Time windows, skills matching, the maximum # patient visited by a caregiver	MILP	Adaptive large neighborhood search in a multi-directional local search framework	Uncertain service time, scenario-based	Daily

*SOO: Single-Objective Optimization, MOO: Multi-Objective Optimization, MIP: Mix-Integer Programming, MILP: Mix-Integer Linear Programming, IP: Integer Programming, CCP: Chance-Constrainted Programming, SPR: Stochastic Programming with Recourse, SP: Stochastic Programming, MDP: Markov decision process. ↑ means to maximize the objective function, others are minimized.

TABLE 2.1 – A summary of the literature in the HHCRSP Part 4

Chapter 3

Weekly routing and scheduling problems

As discussed in Chapters 1 and 2, we highlighted the importance of satisfying the main stakeholders' needs, which include reducing costs, ensuring timely service, and avoiding burnout. There remains a gap in the consideration of the multi-objective home health care planning problem with heterogeneous caregivers. This chapter aims to bridge this gap by exploring two key research questions, Q1 and Q2: How can optimal weekly routes and schedules be created to accommodate the needs of three different stakeholders? And what is the ideal number of each type of caregiver to hire to effectively manage fluctuating task volumes? The structure of this chapter is as follows: Section 3.1 outlines the problem and introduces its formulation through Mixed-integer Linear Programming. Section 3.2 presents the solution methodology, i.e., a multi-objective optimization method. In Section 3.3, we conduct computational experiments based on real-world cases and discuss the results. Section 3.4 provides a conclusion to the chapter.

Based on this work, a paper has been submitted for consideration in the journal Computers & Operations Research (currently under review). We also plan a presentation at the International Conference on Control, Automation and Diagnosis (IEEE IFAC ICCAD 2024).

3.1 Problem statement

In this section, we describe some assumptions and characteristics of the HHCRSP in our case and then establish an MILP model for the problem.

3.1.1 Definition and assumption

This study aims to make routes and schedules for caregivers over a given planning horizon. We assume that travel time is proportional to the Euclidean distance between two nodes. The principle of the triangular inequality holds in terms of travel time and travel distance. Internal caregivers utilize company-provided cars. The calculation of their travel expenses is based on multiplying the social costs per kilometer by the travel distance. For external caregivers, the travel costs are correlated with the distance from each patient to the caregiver's residence.

Patients tend to be visited by a group of familiar caregivers. To ensure the continuity of care for patients, the number of different caregivers visiting a single patient is kept. Patients may receive services one or several times a day, and these services could be needed for one or several days within one week. In this paper, these services are referred to as "jobs". Therefore, some jobs required by the same patient are located in the same place. In France, the charges for care associated with these dependent jobs fall into two primary categories: BSI (Nursing Care Plan) and AMX (Nursing Medical Procedures).

The severity of a patient's condition influences the complexity of the caregiver's work. In France, once elderly individuals register with an HHC company, they are assessed based on six levels of loss of autonomy, named the GIR (iso-resource group), which indicates their level of dependency. GIR 1 represents the strongest dependency level, while GIR 6 represents the weakest. A lower GIR level, indicating poorer health and greater difficulty in handling the patient, typically corresponds to higher job complexity for caregivers. The elderly classified under GIR 5 and 6 are considered to have autonomy and are not entitled to the Personal Autonomy Allowance (APA) reserved for GIR 1 to 4. In the real-life data that is provided by an HHC company, patients with GIR 5 and 6 are not admitted. Therefore, in our study, we only consider GIR 1 to 4.

Internal caregivers depart from the facility, while external caregivers leave from their domiciles. After finishing their last job, all caregivers return to the facility to summarize their day's work. Consequently, we should consider our problem as a combination of a multi-depot routing and a scheduling problem.

Internal and external caregivers are paid in different ways. The former are hired by the HHC company with a basic salary. If their daily working hours exceed the contract working time, they receive overtime pay. External caregivers are paid based on the number of jobs they complete, and their nursing compensation is calculated through the costs of BSI and AMX. Local regulations often limit the number of caregivers of each type. A constraint has been established to control this ratio accordingly.

Caregivers will start their services immediately upon arrival if they reach the location within or after the time windows. If they arrive early, they will wait until the earliest times of time windows are reached before starting services. The service lasts for a specific period (service time). After completing a job, they immediately proceed to the next job or return to the facility.

Three objectives are optimized. The first objective is to minimize travel and staff costs. By minimizing the wages of both types of caregivers, we aim to achieve better utilization of both types. The second objective is to minimize penalties, which serve as a measure of the satisfaction of caregivers and patients. Penalties are incurred if the caregivers' arrival or departure times fall outside the time windows. A high penalty value could lead to excessive waiting times for caregivers or delays in service for patients. The final objective is to minimize workload differences between each pair of caregivers to guarantee fairness. The workload is measured in terms of the daily service times spent with patients, the travel times, and the complexity of jobs which is evaluated based on the GIR.

3.1.2 Mathematical model

In this section, a mathematical model of mid-term multi-depot routing and scheduling problems in HHC has been formulated. From the perspective of graph theory: let $G = (V, A)$, where $V = \{0, n+o\} \cup O \cup N$ is the vertex set, representing the depot, external caregivers' departure points and the jobs required by patients. All caregivers will return to $n+o$ after finishing their service for the day. The depot is simultaneously represented by the two vertices, 0 and $n+o$, which means 0 and $n+o$ have the same location. $A = \{(i,j) \in V, i \neq j\}$ is the set of arcs. Let $S \subset V$ be an arbitrary subset of vertices. The in-arcs and out-arcs of S are defined as $\delta^-(S) = \{(i,j) \in A : i \notin S, j \in S\}$ and $\delta^+(S) = \{(i,j) \in A : i \in S, j \notin S\}$, respectively. It is standard to define $\delta(i) := \delta(\{i\})$ for singleton sets $S = \{i\}$ (similarly, $\delta^+(i)$ and $\delta^-(i)$).

We assume caregivers leave the depot at the initial time, time 0, moving from point i to point j. We consider $t_{0i} \leq e_i^d$ to ensure caregivers arrive at the first job before its scheduled time window starts. The goal is to develop a cost-effective tour that visits each vertex once, complying with the constraints of a subset of arcs, R⊆A.

Table 3.1 shows the notations used for the mathematical model.

TABLE 3.1 – Notations.

Notation	Definition
Sets	
V	set of all vertices
N	set of all jobs
DC	set of external caregivers' departure locations
O	set of starting location of all caregivers, $O = \{0, DC\}$
SL_p	set of jobs required by the same patient p on one day
K	set of all caregivers, K_{in} denotes the set of internal caregivers, while K_{ex} means the set of external caregivers, $K = \{K_{in}, K_{ex}\}$
G	set of interval divided by arrival time
H	sef of interval divided by departure time
C	set of classifications of GIR, C={1, 2, 3, 4}
D	set of planning days
Parameters	
i,j	index of vertices
k	index of caregivers
p	index of patients
d	index of planning days
c	index of classifications of GIR
o	$o = \|O\|$
n	$n = \|N\|$
t_{ij}	travel distance between i and j
s_i^d	service time of patient i on day d
F_i^d	if i is visited on day d
e_i^d, l_i^d	time window of job i

Notation	Definition
α_h	degree coefficient if arrival time is located at $g^{th} \in G$ interval
β_g	degree coefficient if departure time is located at $h^{th} \in H$ interval
GIR_i^c	$4 - c$ if GIR level of job i is c
WT	daily contract working time for internal caregivers
NC	limited number of different caregivers served one patient
MWT	daily maximum working time for caregivers
φ_1, φ_1	coefficients for working time difference and work complexity difference
$\omega_t, \omega_{ov}, \omega_s$	unit costs for travel distance, overtime and fixed salary for internal caregivers, respectively
ω_v	reciprocal of speed
R_1, R_2	ratio of the external caregivers to the internal caregivers, they could be offered by local rules
TE_{ik}	travel cost for the external caregiver k to job i
BSI_i	BSI cost for job i
AMX_i^d	AMX cost for job i on day d
M	a positive large number
Decision variables	
x_{ijk}^d	binary decision variable, 1 if caregiver k moves from i to j on day d, 0 otherwise
y_{ik}^d	binary decision variable, 1 if job i is served by caregiver k on day d, 0 otherwise
OV_k^d	overtime for internal caregiver k on day d
a_{ik}^d	arrival time that caregiver k arrives at patient i on day d
b_{ik}^d	departure time that caregiver k leaves from patient i on day d
pA_{ik}^d, pB_{ik}^d	continuous decision variable, penalties that arrival time and departure time are outside of time windows
wA_{ik}^d, wB_{ik}^d	auxiliary variables, continuous

Notation	Definition
${u_{ig}}^d, {v_{ih}}^d$	binary decision variable, 1 if arrival (departure) time at patient i is located at g^{th} (h^{th}) interval on day d, 0 otherwise
r_i^d	binary decision variable, 1 if caregiver arrives after e_i^d
w_k^d	total working time of caregiver k on day d, including travel time and service time
gN_k^{cd}	the number of jobs with GIR level c that caregiver k serves on day d
q_{ik}	binary variable, if caregiver k serve patient i during planning horizon
qN_p	the number of different caregivers that visit patient p

$$f_1 = \min \omega_t \sum_{d \in D} \sum_{k \in K_{in}} \sum_{i \in \{0,N\}} \sum_{j \in \{0,N\}} t_{ij} x_{ijk}^d + \sum_{d \in D} \sum_{k \in K_{ex}} \sum_{i \in N} TE_{ik} y_{ik}^d + \omega_{ov} \sum_{d \in D} \sum_{k \in K_{in}} OV_k^d + \omega_s \max_{d \in D} (\sum_{k \in K_{in}} (1 - x_{0,n+o,k}^d)) + \sum_{d \in D} \sum_{k \in K_{ex}} \sum_{i \in N} (BSI_i + AMX_i^d) y_{ik}^d \tag{3.1}$$

$$f_2 = \min \sum_{i \in N} \sum_{k \in K} \sum_{d \in D} (pA_{ik}^d + pB_{ik}^d) \tag{3.2}$$

$$f_3 = \min \sum_{k \in K} \sum_{k' \in K : k' \geq k} (\varphi_1 \sum_{d \in D} \left| w_k^d - w_{k'}^d \right| + \varphi_2 \sum_{c \in C} \sum_{d \in D} \left| gN_k^{cd} - gN_{k'}^{cd} \right|) \tag{3.3}$$

s.t.

$$\sum_{k \in K} \sum_{j \in \delta^+(i)} x_{ijk}^d = F_i^d, \forall i \in N \cap O, \forall d \in D \tag{3.4}$$

$$\sum_{i \in DC} \sum_{j \in \delta^+(i)} x_{ijk_{ex}}^d + \sum_{j \in \delta^+(i)} x_{0jk_{in}}^d \leq 1, \forall k_{in} \in K_{in}, \forall k_{ex} \in K_{ex}, \forall d \in D \tag{3.5}$$

$$\sum_{i \in \delta^-(n+o)} x_{i,n+o,k}^d = 1, \forall k \in K, \forall d \in D \tag{3.6}$$

$$\sum_{i\in\delta^-(j)} x_{ijk}^d = \sum_{i\in\delta^-(j)} x_{jik}^d, \forall j \in N, \forall k \in K, \forall d \in D \tag{3.7}$$

$$y_{ik}^d = \sum_{j\in\delta^+(i)} x_{ijk}^d, \forall i \in N, \forall k \in K, \forall d \in D \tag{3.8}$$

$$y_{ik}^d \leq q_{ik}, \forall i \in N, \forall k \in K, \forall d \in D \tag{3.9}$$

$$q_{ik} \leq \sum_{d\in D} y_{ik}^d, \forall i \in N, \forall k \in K \tag{3.10}$$

$$qN_p = \sum_{i\in SL_p} \sum_{k\in K} q_{ik}, \forall p \in P \tag{3.11}$$

$$qN_p \leq NC, \forall p \in P \tag{3.12}$$

$$w_k^d = \sum_{i\in N} \sum_{j\in\delta^+(i)} x_{ijk}^d s_i^d + \omega_v \sum_{i\in N} \sum_{j\in\delta^+(i)} x_{ijk}^d t_{ij}, \forall k \in K, \forall d \in D \tag{3.13}$$

$$gN_k^{cd} = \sum_{i\in N} \sum_{j\in\delta^+(i)} x_{ijk}^d GIR_i^c, \forall c \in C, \forall k \in K, \forall d \in D \tag{3.14}$$

$$MWT \geq w_k^d, \forall k \in K, \forall d \in D \tag{3.15}$$

$$b_{ik}^d = 0, \forall i \in O, \forall k \in K, \forall d \in D \tag{3.16}$$

$$b_{ik}^d = \max(e_i^d, a_{ik}^d) + s_i^d, \forall i \in N, \forall k \in K, \forall d \in D \tag{3.17}$$

$$a_{jk}^d = x_{ijk}^d(b_{ik}^d + t_{ij}), \forall (i,j) \in A, \forall k \in K, \forall d \in D \tag{3.18}$$

$$b_{ik}^d \geq a_{ik}^d + s_i^d, \forall i \in N, \forall k \in K, \forall d \in D \tag{3.19}$$

$$b_{ik}^d \geq e_i^d + s_i^d, \forall i \in N, \forall k \in K, \forall d \in D \tag{3.20}$$

$$b_{ik}^d \leq a_{ik}^d + s_i^d + (1 - r_i^d) * M, \forall i \in N, \forall k \in K, \forall d \in D \tag{3.21}$$

$$b_{ik}^d \leq e_i^d + s_i^d + r_i^d * M, \forall i \in N, \forall k \in K, \forall d \in D \tag{3.22}$$

$$b_{ik}^d + t_{ij} \leq a_{jk}^d + (1 - x_{ijk}^d) * M, \forall (i,j) \in A, \forall k \in K, \forall d \in D \tag{3.23}$$

$$b_{ik}^d + t_{ij} \geq a_{jk}^d - (1 - x_{ijk}^d) * M, \forall (i,j) \in A, \forall k \in K, \forall d \in D \tag{3.24}$$

$$OV_k^d \geq w_k^d - WT, \forall k \in K_{in}, \forall d \in D \tag{3.25}$$

$$R_1 \geq \sum_{i \in DC} \sum_{d \in D} \sum_{k \in K_{ex}} (1 - x_{i,n+o,k}^d) / \sum_{d \in D} \sum_{k \in K_{in}} (1 - x_{0,n+o,k}^d) \geq R_2 \tag{3.26}$$

$$pA_{ik}^d = \begin{cases} \alpha_0, a_{ik}^d \leq e_i^d - 30 \\ \alpha_1, e_i^d - 30 < a_{ik}^d \leq e_i^d - 15 \\ \alpha_2, e_i^d - 15 < a_{ik}^d \leq e_i^d \\ \alpha_3, e_i^d < a_{ik}^d \leq l_i^d \\ \alpha_4, l_i^d < a_{ik}^d \end{cases} , if\ x_{ijk}^d = 1, \forall i \in N, \forall k \in K, \forall d \in D \tag{3.27}$$

$$pB_{ik}^d = \begin{cases} \beta_0, b_{ik}^d \leq l_i^d \\ \beta_1, l_i^d < b_{ik}^d \leq l_i^d + 15 \\ \beta_2, l_i^d + 15 < b_{ik}^d \leq l_i^d + 30 \\ \beta_3, l_i^d + 30 < b_{ik}^d \end{cases} , if\ x_{ijk}^d = 1, \forall i \in N, \forall k \in K, \forall d \in D \tag{3.28}$$

$$\sum_{g \in G} u_{ig}^d = 1, \forall i \in N, \forall k \in K, \forall d \in D \tag{3.29}$$

$$a_{ik}^d \leq (e_i^d - 30) * u_{i0}^d + (e_i^d - 15) * u_{i1}^d + e_i^d * u_{i2}^d + l_i^d * u_{i3}^d + M * u_{i4}^d, \forall i \in N, \forall k \in K, \forall d \in D \tag{3.30}$$

$$a_{ik}^{d} \geq (e_{i}^{d} - 30) * u_{i1}^{d} + (e_{i}^{d} - 15) * u_{i2}^{d} + e_{i}^{d} * u_{i3}^{d} + l_{i}^{d} * u_{i4}^{d}, \forall i \in N, \forall k \in K, \forall d \in D \tag{3.31}$$

$$wA_{ik}^{d} = \sum_{g \in G} \alpha_{g} u_{ig}^{d}, \forall i \in N, \forall k \in K, \forall d \in D \tag{3.32}$$

$$pA_{ik}^{d} \leq wA_{ik}^{d} + M * (1 - \sum_{j \in \delta^{+}(i)} x_{ijk}^{d}), \forall i \in N, \forall k \in K, \forall d \in D \tag{3.33}$$

$$pA_{ik}^{d} \geq wA_{ik}^{d} - M * (1 - \sum_{j \in \delta^{+}(i)} x_{ijk}^{d}), \forall i \in N, \forall k \in K, \forall d \in D \tag{3.34}$$

$$pA_{ik}^{d} \leq M * \sum_{j \in \delta^{+}(i)} x_{ijk}^{d}, \forall i \in N, \forall k \in K, \forall d \in D \tag{3.35}$$

$$\sum_{h \in H} v_{ih}^{d} = 1, \forall i \in N, \forall d \in D \tag{3.36}$$

$$b_{ik}^{d} \leq l_{ik}^{d} * v_{i0}^{d} + (l_{ik}^{d} + 15) * v_{i1}^{d} + (l_{ik}^{d} + 30) * v_{i2}^{d} + M * v_{i3}^{d}, \forall i \in N, \forall k \in K, \forall d \in D \tag{3.37}$$

$$b_{ik}^{d} \geq l_{ik}^{d} * v_{i1}^{d} + (l_{ik}^{d} + 15) * v_{i2}^{d} + (l_{ik}^{d} + 30) * v_{i3}^{d}, \forall i \in N, \forall k \in K, \forall d \in D \tag{3.38}$$

$$wB_{ik}^{d} = \sum_{h \in H} \beta_{h} v_{ih}^{d}, \forall i \in N, \forall k \in K, \forall d \in D \tag{3.39}$$

$$pB_{ik}^{d} \geq wB_{ik}^{d} - M * (1 - \sum_{j \in \delta^{+}(i)} x_{ijk}^{d}), \forall i \in N, \forall k \in K, \forall d \in D \tag{3.40}$$

$$pB_{ik}^{d} \leq wB_{ik}^{d} + M * (1 - \sum_{j \in \delta^{+}(i)} x_{ijk}^{d}), \forall i \in N, \forall k \in K, \forall d \in D \tag{3.41}$$

$$pB_{ik}^{d} \leq M * \sum_{j \in \delta^{+}(i)} x_{ijk}^{d}, \forall i \in N, \forall k \in K, \forall d \in D \tag{3.42}$$

$$x_{ijk}^d, y_{ik}^d, r_i^d, u_{ig}^d, v_{ih}^d, q_{ik}, qN_p \in \{0,1\}, \forall (i,j) \in A, \forall k \in K, \forall d \in D, \forall g \in G, \forall h \in H, \forall p \in P \tag{3.43}$$

$$a_{ik}^d, b_{ik}^d, pA_{ik}^d, pB_{ik}^d, wA_{ik}^d, wB_{ik}^d, OV_k^d, w_k^d, gN_k^{cd} \geq 0, \forall i \in N, \forall k \in K, \forall d \in D \tag{3.44}$$

The objective function (3.1) minimizes the operational cost composed of the travel cost and the staff costs. The second objective function (3.2) minimizes the penalty. If the penalty is smaller, more patients can receive service on time and more caregivers do not need to wait too long. The differences of workload of each pair of caregivers are optimized through (3.3). Constraints (3.4) show each job can be visited exactly once in one day. Constraints (3.5) and (3.6) indicate internal caregivers start from the facility while the external ones start from their own homes, and they all return to the facility at the end of service to summarize their work. Constraints (3.7) show the flow of each caregiver. Constraints (3.8) define y_{ik}^d. (3.9)-(3.12) limit the maximum number of different caregivers serving one patient during the planning horizon. The workload balance is defined by constraints (3.13)-(3.14). The daily maximum working time for each caregiver is limited by constraints (3.15). Constraints (3.16) mean each caregiver leaves from the depot or their home at time 0. If the caregiver arrives before e_i^d, the service will not start before reaching e_i^d; if the caregiver arrives after e_i^d, the service will start immediately. The caregiver will leave immediately after serving for s_i^d. The departure time that caregivers leave each job's location can be calculated as constraints (3.17). Constraints (3.18) mean the arrival time when caregiver k travels from job i to j. (3.16)-(3.18) are linearized by constraints (3.19)-(3.24). Constraints (3.25) calculate the overtime. Constraint (3.26) ensures that the ratio of external caregivers to internal ones used during the planning horizon must be within a certain range. The formulas (4.3) and (4.4) define the second objective. Different penalty values α_g or β_h are assigned to the intervals $[0, e_i^d - 30]$, $(e_i^d - 30, e_i^d - 15]$, $(e_i^d - 15, e_i]$, $(e_i^d, l_i]$, $(l_i^d, l_i^d + 15]$, $(l_i^d + 15, l_i^d + 30]$, $(l_i^d + 30, \infty]$. In other words, we have two loose time windows $[e_i^d - 30, l_i^d + 30]$ and $[e_i^d - 15, l_i^d + 15]$ and one tight time window $[e_i^d, l_i^d]$. Discrete penalties can prevent a few patients from suffering from large delays. The linearization of (4.3) and (4.4) are expressed by (3.29)-(3.42).

Constraints (3.43) and (3.44) show the domain of variables.

To solve this MILP model, we employ the augmented ε-constraint method - AUGMECON (for a comprehensive understanding, refer to Mavrotas, 2009 and Govindan et al., 2023). By varying the parameters on the right-hand side of the constrained objective functions (denoted by $e_p, p \in \{1, .., P\}$, where p is the index of the objective functions), the Pareto optimal solutions for the problem are derived. The range of e_p can be determined from the payoff table using the lexicographic method. The range of each objective function is evenly divided into q_p intervals, resulting in e_p variations across $q_p + 1$ grid points. We optimize the first objective function and treat the other objective functions as constraints. The model can be rewritten as follows:

$$f_1 - eps \cdot (s_2/r_2 + s_3/r_3), \tag{3.45}$$

s.t.

$$\sum_{i \in N} \sum_{k \in K} \sum_{d \in D} (pA_{ik}^d + pB_{ik}^d) + s_2 = e_2, \tag{3.46}$$

$$\sum_{k \in K} \sum_{k' \in K : k' \geq k} (\varphi_1 \sum_{d \in D} \left| w_k^d - w_{k'}^d \right| + \varphi_2 \sum_{c \in C} \sum_{d \in D} \left| gN_k^{cd} - gN_{k'}^{cd} \right|) + s_3 = e_3, \tag{3.47}$$

$$s_1 \geq 0, s_2 \geq 0, s_3 \geq 0, \tag{3.48}$$

$$(3.4) - (3.44)$$

where eps is a very small constant, equal to 10^{-6}; s_2 and s_3 are non-negative surplus variables; and r_2 and r_3 represent the ranges of (3.2) and (3.3) obtained from the payoff table. The number of sub-problems is $(q_2 + 1) \times (q_3 + 1)$.

3.2 Solving method

Due to the computational complexity, we develop an ϵ-greedy Large Neighbourhood Search (ϵ-gLNS) embedded in a Multi-Directional Local Search algorithm to solve the multi-objective optimization problem. We adapt the traditional operators and design new ones in ϵ-gLNS considering the characteristics of the proposed problem. First, the basic framework and our improvement to the MDLS are introduced in Section 3.2.1. Then, in Section 3.2.2, the ϵ-gLNS primarily innovates by customizing operators based on specific characteristics and adjusting the number of removal nodes using ϵ.

3.2.1 Multi-directional local search algorithm (MDLS)

The MDLS was first proposed by (Tricoire, 2012). It functions by performing a local search on each objective (direction) and iteratively improving the non-dominated front (F). Each local search works independently, without considering the importance of the objectives. The framework is shown in Algorithm 1. The Savings algorithm is a constructive heuristic for constructing an initial solution $\mathbf{x}$. Instead of randomly selecting a solution in the original paper (Tricoire, 2012), one solution is selected from F based on crowding distance, which is inspired by the NSGA-II (Deb et al., 2002). The crowding distance of a point i in F can be regarded as the perimeter of a hypercube surrounded by two adjacent points $i-1$ and $i+1$. The boundary points are assigned to a very large value. A solution with a larger crowding distance is more likely to be selected. Subsequently, the ϵ-gLNS is employed to improve $\mathbf{x}$. To promote diversity, more than one solution is stored in G for each direction. The Deb non-dominated sorting method is utilized to maintain the non-dominated front F after each iteration. F and G are saved as an ordered list to reduce the time required for the non-dominated sorting.

Algorithm 1 Improved Multi-Directional Local Search for Three Objectives

Input: a initial solution $\mathbf{x}$, the non-dominated front F only including $\mathbf{x}$

Output: F including a pool of non-dominated solutions

```
repeat
    x ← select_a_solution(F)
    G ← ∅
    for k ← 1 to 3 do
        G ← G ∪ ϵ − gLNS_k(x)
    end for
    F ← Deb_nondominated_sorting(F, G)
until meet stopping criterion
```

3.2.2 ϵ-greedy Large Neighborhood Search

The Large Neighbourhood Search (LNS) is used to find a new solution in a large neighborhood of the current solution, and then the new solution can be accepted by a certain acceptance criterion. After several iterations, the solution and its objective are updated. As shown in Fig. 3.1, the solution of our problem can be structured as a sequence of ordered nodes representing jobs over one week. A destroy operator is employed to remove a number of nodes from the solution. These nodes can be reinserted to generate a new solution via a repair operator.

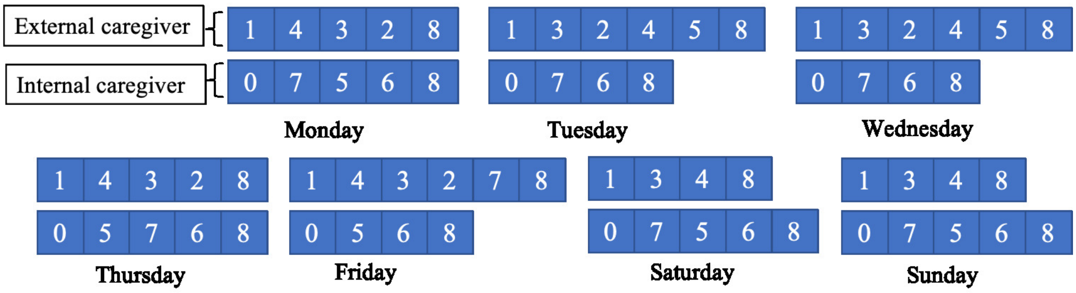


FIGURE 3.1 – Solution structure.

In comparison to the LNS we mentioned above, a new solution can be created by different types of operators in each iteration. We develop various operators based on the previous work by (Ropke and Pisinger, 2006), and some of these are specifically tailored to our constraints and objectives. Furthermore, the number of removal nodes can be varied, thereby extending the neighborhood to some extent, when the best-found solution has not been updated for a certain period. Finally, a certain number of best-found solutions obtained in each iteration are saved.

As demonstrated in Algorithm 2, at the beginning of the loop, one destroy operator and one repair operator are randomly selected to construct a new solution $\mathbf{x}_{\text{new}}$ from the current solution $\mathbf{x}_{\text{cur}}$. $\mathbf{x}_{\text{new}}$ can be accepted if its objective value $f(\mathbf{x}_{\text{new}})$ is better than the current objective value $f(\mathbf{x}_{\text{cur}})$. The best-found solution can be updated if its objective value $f(\mathbf{x}_{\text{bf}})$ is worse than $f(\mathbf{x}_{\text{new}})$. According to record-to-record acceptance criterion (Dueck, 1993), $\mathbf{x}_{\text{cur}}$ can also be replaced by $\mathbf{x}_{\text{new}}$ if $f(\mathbf{x}_{\text{new}})$ is better than $(1+d)$ times $f(\mathbf{x}_{\text{bf}})$. After a period, the algorithm may fall into local optimum, which is judged by $\mathbf{x}_{\text{cur}}$ has not been updated after T consecutive iterations. We then increase the number of removal nodes sl to a larger value with a probability of ϵ, where $\epsilon = 1/k$ and k denotes the number of iterations currently completed.

Algorithm 2 Improved Large Neighborhood Search

Input: an initial solution $\mathbf{x}$, destroy operators $destroy_i$, $repair_j$, $iter_{\text{LNS}}$, N_{s}^{r},N_{l}^{r}, deviation d,sl, T

Output: a pool of solutions $\mathcal{P}$

$\mathcal{P} \leftarrow \emptyset$, $\mathbf{x}_{\text{bf}} \leftarrow \mathbf{x}$, $\mathbf{x}_{\text{cur}} \leftarrow \mathbf{x}$, $sl \leftarrow small$
for $k \leftarrow 0$ to $iter_{\text{LNS}}$ **do**
  randomly select one destroy operator $destroy_i$ and one repair operator $repair_j$
  $\mathbf{x}_{\text{new}} \leftarrow repair_j(destroy_i(\mathbf{x}_{\text{cur}}, sl))$
  if $sl = N_{\text{l}}^{\text{r}}$ **then**
    $sl \leftarrow N_{\text{s}}^{\text{r}}$
  end if
  if $f(\mathbf{x}_{\text{new}}) < f(\mathbf{x}_{\text{cur}})$ **then**
    $\mathbf{x}_{\text{cur}} \leftarrow \mathbf{x}_{\text{new}}$
    if $f(\mathbf{x}_{\text{new}}) < f(\mathbf{x}_{\text{bf}})$ **then**
      $\mathbf{x}_{\text{bf}} \leftarrow \mathbf{x}_{\text{new}}$
    end if
  else if $f(\mathbf{x}_{\text{new}}) \leq f(\mathbf{x}_{\text{bf}}) + d * f(\mathbf{x}_{\text{bf}})$ **then**
    $\mathbf{x}_{\text{cur}} \leftarrow \mathbf{x}_{\text{new}}$
  end if
  if during T consecutive iterations, $f(\mathbf{x}_{\text{new}})$ remain the same value **then**
    $\mathcal{P} \leftarrow \mathcal{P} \cap \mathbf{x}_{\text{bf}}$
    $p \leftarrow$ uniform random number between 0 and 1
    if $p \leq \epsilon$ **then**
      $sl \leftarrow N_l^{\text{r}}$
    else
      $sl \leftarrow N_s^{\text{r}}$
    end if
  end if
end for

Destroy operators

This section describes six destroy operators removing sl nodes from the current solution. The nodes, along with their corresponding planning days, are saved in a removal list U. The destroy operators end up with the removal list and a current solution minus the nodes in U.

Random destroy operator: This method involves randomly removing sl nodes from the current solution in order to diversify the search scope. It can be implemented easily and tends to run faster than the other methods.

Worst destroy operator: The pseudo-code of this method is presented in Algorithm 3. The objective value $f(\mathbf{x}_{\text{cur}})$ is calculated at first. $cost_i$ denotes the decrease of $f(\mathbf{x}_{\text{cur}})$ if removing node i from $\mathbf{x}_{\text{cur}}$. Nodes with higher $cost_i$ are more likely to be selected. The key of this operator is to select the nodes that have the greatest potential to reduce the objective value and appear to be mispositioned in the solution. If the solution does not satisfy constraints (3.26), nodes will be reselected until U includes sl nodes.

Algorithm 3 Worst Destroy Operator

Input: an initial solution $\mathbf{x}_{\text{cur}}$, sl

Output: removal list U, $\mathbf{x}'_{\text{cur}}$

```
while sl > 0 do
    L contains all nodes of the solution
    for node i in x_cur do
        calculate the difference cost_i between removing i and not removing i
    end for
    sort L in descending order of cost_i
    random number y in interval [0,1)
    u ← L[y|L|]
    x'_cur ← remove(x_cur, u)
    U ← U ∪ u
    sl ← sl − 1
end while
```

Relatedness destroy operator: A relatedness destroy operator tends to select nodes that are similar. Reinserting related nodes is more likely to result in the development of a new and improved solution. The relatedness can be regarded as a close distance between two nodes, similar time windows, the same job on different days, and jobs with the same GIR levels. Algorithm 4 provides the pseudo-code for this operator. A node j is randomly selected from the removal list. For each node i in the solution, relatedness $R_i(i,j)$ is calculated. i with a higher relatedness indicates that i is more similar to j, and is more likely to be removed.

For the local search used to improve the first objective (3.1), the relatedness is defined by close geographical locations, which is computed by:

$$R_i^{\mathrm{a}}(i,j) = \frac{1}{t_{ij}/t_{\max} + v}, \tag{3.49}$$

where t_{ij} denotes the travel cost between node i and j, $t_{\max}$ denotes the largest travel cost among all pairs of i and j. The relatedness for the second objective is calculated by:

$$R_i^{\mathrm{b}}(i,j) = \frac{1}{(|e_i - e_j| + |l_i - l_j|)/tw_{\max} + v}, \tag{3.50}$$

where e_i and l_i denote the earliest time and the latest time of the time window; and $tw_{\max}$ means the length of the longest time window. If i and j are in the same route, v is equal to 0. i and j are the nodes on the same planning day.

Algorithm 4 Relatedness Destroy Operator

Input: an initial solution $\mathbf{x}_{\text{cur}}$, sl

Output: removal list U, $\mathbf{x}_{\text{cur}}^{'}$

randomly choose a node u from the $\mathbf{x}_{\text{cur}}$
$\mathbf{x}_{\text{cur}}^{'} \leftarrow remove(\mathbf{x}_{\text{cur}}, u)$
$U \leftarrow U \cup d$
while $sl - 1 > 0$ **do**
    randomly select a node j from U
    L contains all nodes of the solution
    for node i in $\mathbf{x}_{\text{cur}}^{'}$ **do**
        calculate $R_i(i, j)$
    end for
    sort L by ascending order of $R_i(i, j)$
    random number y in interval [0,1)
    $u \leftarrow L[y|L|]$
    $\mathbf{x}_{\text{cur}}^{'} \leftarrow remove(\mathbf{x}_{\text{cur}}, u)$
    $U \leftarrow U \cup u$
    $sl \leftarrow sl - 1$
end while

Related job destroy operator: Identical jobs performed on different days are removed to enhance the continuity of care. For example, if we randomly select job 4, it can be removed from the solutions of sl planning days. If the chosen job is available for fewer than sl planning days, another job is selected until sl nodes are placed in the removal list U.

Related GIR level destroy operator: Algorithm 5 shows the procedures to remove sl nodes based on the related GIR level. This method removes jobs with the same GIR level within the same planning day in order to achieve a better balance in work complexity, thereby improving the objective (3.3).

Route destroy operator: If we reduce one internal caregiver and transfer the patients served by the internal caregiver to an external caregiver, it may significantly reduce the first objective function due to the reduction in fixed salary costs. Therefore, we employ this operator when optimizing the first objective function. One route assigned to an internal caregiver and another assigned to an external caregiver are randomly chosen and removed from $\mathbf{x}_{\text{cur}}$.

Algorithm 5 Related GIR Level Destroy Operator

Input: an initial solution $\mathbf{x}_{\text{cur}}$, sl

Output: removal list U, $\mathbf{x}_{\text{cur}}$

randomly choose a planning day d

integrate all nodes on the day d in one list L^{d}

while $sl - 1 > 0$ **do**

  randomly select a node j from L^{d}

  for node i in $\mathbf{x}_{\text{cur}}$ **do**

    if $GIR_j = GIR_i$ **then**

      remove i from $\mathbf{x}_{\text{cur}}$

      $U \leftarrow U \cup i$

      $sl \leftarrow sl - 1$

    end if

    if $sl - 1 <= 0$ **then**

      break

    end if

  end for

end while

Repair operators

Three repair operators are proposed, building upon the previous work by Ropke and Pisinger (2006). Each node in the removal list generated by destroy operators can be reinserted into the current solution. However, the reinsertion of a removed node into its original planning day's route may lead to infeasible solution. After repairing a solution, the following actions are taken: (a) new routes are created, (b) the schedules for the caregivers to run these routes are updated, and (c) the feasibility of the current solution is checked. In line with the mathematical model, the following constraints will be verified:

— constraints (3.15) related to the maximum daily working time;

— constraints (3.12) related to the maximum number of caregivers serving one patient;

— constraints (3.26) related to the ratio between two types of caregivers.

If the solution violates the constraints (3.15), it cannot be accepted, and we retain the seed from the previous iteration. If constraints (3.12) and (3.26) cannot be satisfied, a very large violation penalty will be added to the objective values to ensure that a feasible solution can be found through the use of the repair operators. Other constraints are satisfied automatically in the construction of routes.

Random repair operator: This method involves randomly choosing an insertion position for each node in the removal list. It helps the algorithm explore a larger solution space.

Greedy repair operator: Algorithm 6 outlines the greedy repair operator, which begins by inserting each node i into every possible position $j \in J$ of the solution. The objective difference Δf_i^j between inserting and not inserting node i is then calculated. The cheapest insertion position for node i, denoted by c_i, is identified. Ultimately, the node i and its position j that yield the minimum c_i are selected for insertion, and this process continues until the removal list is empty.

Algorithm 6 Greedy Repair Operator

Input: $\mathbf{x}'_{\text{cur}}$, a removal list U

Output: $\mathbf{x}_{\text{new}}$

while $|U| > 0$ **do**
  for node i in U **do**
    for inserting position j in $\mathbf{x}'_{\text{cur}}$ **do**
      if $\mathbf{x}'_{\text{cur}}$ violate the constraints **then**
        a very large violation penalty will be added to the objective value
      end if
      calculate the difference Δf_i^j of the objective value between inserting i at the position j and not inserting i at the position j
    end for
    $c_i = \min\limits_{j \in J} \Delta f_i^j$
  end for
  find minimum c_i and its corresponding node i and position j
  $\mathbf{x}_{\text{new}} \leftarrow insert(i, j, \mathbf{x}'_{\text{cur}})$
  remove i from U
end while

Regret repair operator: The greedy method iteratively identifies the node with the minimum cost position. However, nodes that incurred a higher cost in the last iteration have fewer opportunities for insertion, as many routes are already "full". To overcome this, the regret method utilizes foresight in its node selection process. The difference in the cost of inserting node i in its best position and its j^{th}-best position is calculated as $\Delta f_i^j - \Delta f_i^0$. This method then selects the node with the highest sum of regret values, represented by $\sum_{j=1}^{k} (\Delta f_i^j - \Delta f_i^0)$, from the removal list.

3.3 Experiments and results

We implement the proposed algorithms in a real-life case study in France. Our data sets are generated based on information from a French HHC company. They provide data involving the services required by anonymous patients. All experiments are conducted on an AMD EPYC 7702 64-Core Processor at 2 gigahertz, using a single thread. Section 3.3.1 describes how we generate the data sets from a real-life case. In Section 3.3.2, the parameters of the algorithms and models are specified, and three performance indicators are introduced. The augmented ε-constraint method and our proposed method are compared in Section 3.3.3. Management recommendations are presented in Section 3.3.4.

3.3.1 Case study and Datasets

In France, a HHC company offers an array of specialized treatments, including SSIAD (home nursing care services), home-based hospitalization and specialized Alzheimer's care, etc. The SSIAD are fully funded by health insurance funds. They provide home care for elderly individuals and people with disabilities, coordinating with other healthcare professionals and operating 7 days a week when necessary. The care services include hygiene care, preventive care for bedsores, assistance with dressing and undressing, and putting on compression stockings, etc. Their goal is to maintain autonomy, avoid hospitalization, and delay entry into care facilities.

The main roles in the SSIAD include IS (internal caregiver), IL (external caregiver) and IC (nurse coordinator). The IS may receive a salary and an overtime pay from the company, while the IL enters into a contract with the SSIAD, billing only for performed acts (BSI, AMX, and travel costs) and managing the social charges.

The travel cost for an IS is ω_t times the total distance traveled to the jobs' locations. In this paper, the parameter ω_t is assigned a value of 0.76 euros/kilometer, aligned with the aggregated private and social costs of automobile travel (Gössling, Kees, and Litman, 2022). Meanwhile, the travel cost of an IL is determined using the following piecewise function:

$$TE = \begin{cases} 2.5\ euros,\ if\ t_{ij} \leq 4\ km \\ 2.5 + 2 * IK * (t_{ij} - 4)\ euros,\ if\ t_{ij} > 4\ km \end{cases} ,\forall i \in O, j \in N, \tag{3.51}$$

where t_{ij} denotes the travel distance from the location of the IL i to the location of job j, and IK represents the mileage allowance in the plains, with IK set at 0.35 euros.

The IC, responsible for the structure, performs a role similar to the healthcare managers in a hospital setting. They determine the admission and discharge of patients within the service and conduct an initial assessment of home care needs. In charge of human resources and management, the IC creates caregivers' schedules and leads coordination meetings. Their essential task is administrative management, which includes establishing the budget and ensuring the quality of care.

Patients with medical prescriptions can access the care services by contacting a local SSIAD provider. An assessment will be conducted, followed by a proposed intervention schedule.

The service hours at a HHC company are available 7 days a week, with morning sessions from 7:30 to 12:00, afternoon sessions from 13:00 to 15:30, and evening sessions from 17:00 to 19:30, depending on the patient's needs and service availability. In the data set provided by the company, patients' geographical locations are represented by latitude and longitude, along with their GIR levels, their visit days, required sessions, and service times. The patients' available times for each service are not predefined. We define them based on the caregivers' available times. For example, if the service is performed in the morning session, the length of the time window will be greater than the service time required by the patient, with the start time e_i^d and the end time l_i^d ranging between 0 and 270 minutes (for the afternoon session between 330 and 480 minutes; for the evening session between 570 and 720 minutes). If caregivers work two shifts in a day, we assume they take breaks during the period between sessions. Service hours and availability for some patients in the dataset change daily. While certain patients maintain consistent service times on different days of the week, others may have varying schedules. This leads to diverse service times and time windows each day across all patient groups.

When the BSI is applied, patients are billed according to their level of dependency as assessed by caregivers: 13 euros for light care, 18.20 euros for intermediate care, and 28.70 euros for heavy care. The AMX, which includes technical treatments such as blood draws and bandaging, complements the BSI. The base price for an AMX service is 3.15 euros, with more complex procedures rated higher. In this case, it can be inferred that services with greater complexity and longer durations would incur higher fees. The BSI fees for each patient are determined based on their GIR level: patients with a GIR level of 1 or 2 are charged 28.70 euros, those at GIR level 3 are charged 18.20 euros, and those at GIR level 4 are charged 13 euros.

Table 3.2 represents the main information contained in the data set.

TABLE 3.2 – Illustration of the data set.

			Monday					Sunday			
Patient	Location	GIR	BSI	AMX	TW	δ	...	BSI	AMX	TW	δ
P1	lon,lat	4	13	0	[100, 270]	30		13	0	[100, 270]	30
...											
PN	lon,lat	2	28.7	0	[400, 440]	20		0	0	[0, 720]	0

*lon,lat: the longitude and latitude; TW: time window; δ: service time. The last patient PN has no services on Sunday.

Three datasets, denoted as $DS_{0\%}$, $DS_{25\%}$, and $DS_{50\%}$, are generated, each with different numbers of patients with high levels of dependency (GIR levels 1 and 2). The proportions of patients with high levels of dependency in these datasets range from 0% to 50%, increasing in increments of 25%. This study aims to examine how the composition of patients influences the numbers of internal caregivers and external caregivers. Each dataset encompasses seven instances, with varying numbers of jobs, denoted as n. The instance naming follows the "HHC-n×7" convention, where 7 indicates a one-week planning horizon, amounting to a total of $n \times 7$ jobs per instance.

3.3.2 Experimental setup and performance indicators

Each objective function, represented as a summation of various components, is normalized to address discrepancies in unit and scale. f_3 includes working time difference f_{wt} and GIR difference f_{gir}. Values are normalized by dividing them by their respective nominal values, derived from the optimization of each objective individually, referred to as calculating ideal objective vectors (Audet et al., 2021). This also ensures a balanced contribution from all components. The objective function (3.2) can be rewritten as:

$$\min \gamma_1 \frac{f_{\mathrm{wt}}}{f_{\mathrm{wt,nom}}} + \gamma_2 \frac{f_{\mathrm{gir}}}{f_{\mathrm{gir,nom}}}, \tag{3.52}$$

where $f_{\mathrm{wt,nom}}$ and $f_{\mathrm{gir,nom}}$ denote the nominal values for each respective component. The positive weights γ_1 and γ_2 indicate the importance of each component which can be decided by HHC company managers, with the constraint that $\gamma_1 + \gamma_2 = 1$. In our case, γ_1 and γ_2 are both set to 0.5. Compared with the objective function (3.3), $\varphi_1 = \gamma_1 / f_{\mathrm{wt,nom}}$ and $\varphi_2 = \gamma_2 / f_{\mathrm{gir,nom}}$.

Due to the infeasibility of thoroughly examining all potential hyperparameter combinations for the ϵ-gLNS-MDLS, we set the hyperparameter d to 0.13, based on our previous work (Zhao, Wang, and Monteiro, 2023). In that work, we employed Bayesian optimization for efficient exploration of hyperparameter combinations (Snoek, Larochelle, and Adams, 2012). We set $iter_{\mathrm{LNS}}$ to a random value ranging between 100 to 1000, allowing for the optimization of the objective function to varying degrees in each iteration, thereby exploring more possibilities.

The gap between the lower and upper objective bounds is set at 0.10% when using the Gurobi solver to obtain optimal solutions. The number of removal nodes, represented as $N^{\mathrm{r}}_{\mathrm{s}}$ and $N^{\mathrm{r}}_{\mathrm{l}}$, is determined randomly within the range of 1 to 3, the range of 4 and 6, respectively. For instances involving only three and six jobs, we did not impose restrictions on the ratio range (R_1, R_2) between internal caregivers and external caregivers, because only a small number of caregivers is required, which rarely exists in the real world. For larger, real-life size instances, we let $R_1 = 5$ and $R_2 = 0$. Managers have the flexibility to adjust the ratio according to practical circumstances. Other parameters of the model are summarized in Table 3.3. Wages, overtime pay and working hours are determined according to the relevant French regulations.

TABLE 3.3 – Parameters of the model.

Parameter	Value
ω_t	0.76 euros/km
ω_{ov}	2.5 euros/min
ω_s	800 euros/week
ω_v	5 min/km
NC	4
MWT	550
WT	360

Three indicators are used to measure the quality of the Pareto optimal solutions produced by AUGMECON and ϵ-gLNS-MDLS.

The number of Pareto optimal solutions (I_n): It is a cardinality indicator. It helps quantify the effectiveness of each algorithm in producing non-dominated solutions.

The spacing indicator (I_s): It is used to evaluate the distribution and uniformity of the Pareto optimal solutions. A smaller spacing indicator value indicates a more uniform distribution of solutions in the Pareto front. This indicator is computed as follows:

$$I_\mathrm{s} = \sqrt{\frac{1}{|Y|-1}\sum_{j=1}^{|Y|}(\bar{d}-d_j)^2}, \tag{3.53}$$

where Y, the Pareto optimal front, contains I_n objective values. Here, d_j can be calculated by $\min\limits_{y \in Y\setminus\{y^j\}} ||y - y^j||_1$, indicating the l_1 distance of $y^j \in Y$ to the set $Y\setminus\{y^j\}$. $\bar{d}$ denotes the mean of all d_j (Audet et al., 2021).

The hypervolume indicator (I_h): It calculates the union of the volumes enclosed by each objective value and a reference point, assessing both convergence and diversity in multi-objective optimization problems. It can be expressed by the following formula:

$$I_\mathrm{h} = \bigcup\nolimits_{y\in Y} V(y, R), \tag{3.54}$$

where R is the reference point. To facilitate comparisons, objective values are scaled to a range of [0, 1], and R is set to (-1, -1, -1). A larger I_h value indicates a higher quality of the solution set. Computing I_h can be computationally demanding, particularly for high-dimensional objective spaces. A dimension-sweep algorithm is employed for calculating I_h (Fonseca, Paquete, and López-Ibánez, 2006) in the following part.

3.3.3 Performance comparison and trade-off analysis

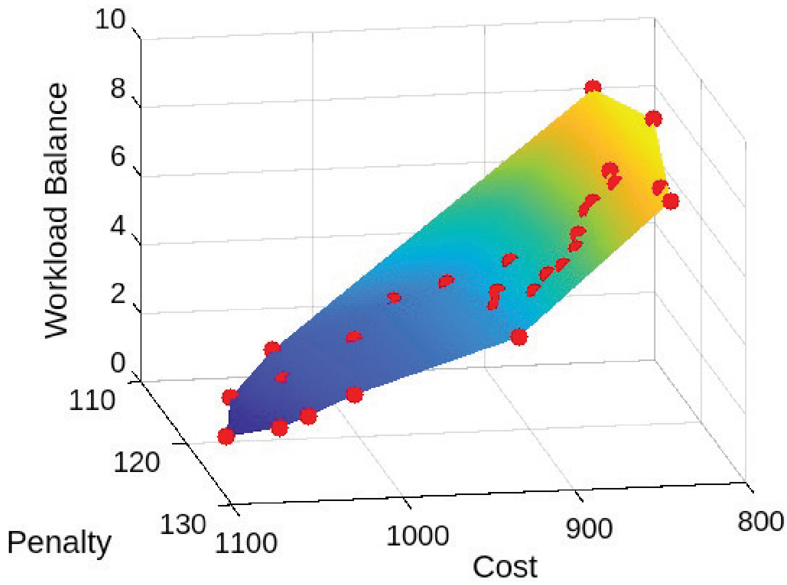
In this subsection, we compare the AUGMECON and our proposed method, ϵ-gLNS-MDLS, in terms of solution quality and solving effectiveness. A trade-off analysis is conducted based on the Pareto fronts. We present the best values derived from five independent runs for our proposed method.

Table 3.4 and 3.5 present various metrics and the extreme points obtained through the AUGMECON and our proposed method. In smaller-sized instances involving three and six jobs, our method shows comparable effectiveness in finding solutions to the AUGMECON, while also offering the advantage of reduced computational time.

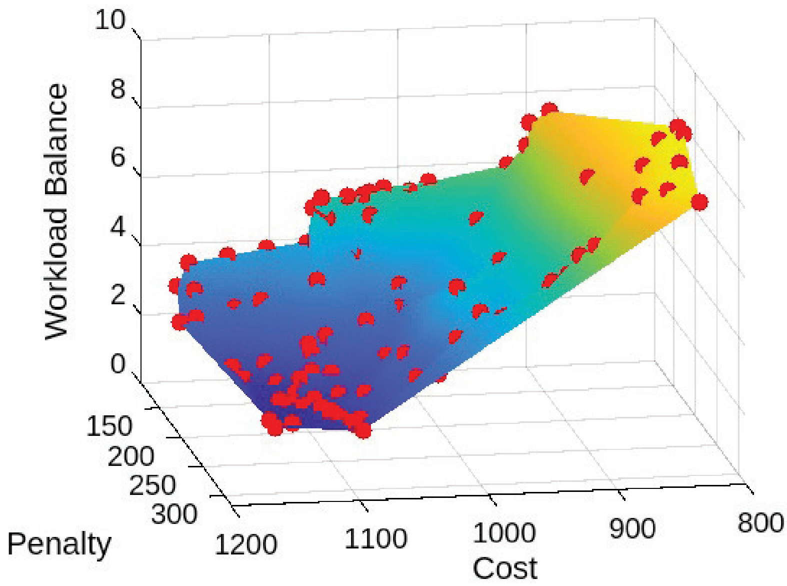
Figure 3.2 provides a detailed graphical representation of the Pareto front approximation. The surface was constructed utilizing the triangulation technique based on the non-dominated points obtained by the AUGMECON (3.2a) and our proposed ϵ-gLNS-MDLS (3.2b). These points are relatively uniformly distributed across the surface for both algorithms, indicating well-distributed and diverse Pareto fronts. The I_n of our method shows a superior performance in comparison to the AUGMECON approach. This enhanced performance is attributed to the larger search space that the proposed method is capable of exploring. In the AUGMECON, e_p is adjusted in each iteration, which resembles a grid-based search and consequently limits the exploratory scope of the solution space.

Our proposed method surpasses the AUGMECON in the speed of identifying potential solutions for the Pareto front. Unlike the AUGMECON, which solves an MILP for optimal results, our approach employs a neighborhood exploration strategy, assessing feasibility and acceptance across three directions (objectives).

It is important to note that as the size of the instance increases, there is a substantial rise in computational time, so attaining an optimal solution presents considerable challenges by utilizing the Gurobi Solver. The AUGMECON is unable to generate solutions within an acceptable time frame for large instances. In contrast, our method remains effective in achieving satisfactory indicators, even when dealing with large-scale instances.



(A) Gurobi solver



(B) ϵ-gLNS-MDLS

FIGURE 3.2 – Simulated Pareto front of HHC-6×7

TABLE 3.4 – Performance indicators.

	$DS_{0\%}$				$DS_{25\%}$				$DS_{50\%}$			
	I_h	I_s	I_n	TCPU	I_h	I_s	I_n	TCPU	I_h	I_s	I_n	TCPU
AUGMECON												
HHC-3×7	0.41	0.08	26	11809	0.44	0.16	8	1598	0.42	0.14	10	1590
HHC-6×7	0.36	0.09	27	10223	0.52	0.06	20	9310	0.35	0.04	21	27605
ϵ-gLNS-MDLS												
HHC-3×7	0.56	0.09	22	265	0.65	0.10	31	290	0.55	0.05	49	312
HHC-6×7	0.56	0.04	88	521	0.47	0.03	94	542	0.52	0.04	98	588
HHC-20×7	0.35	0.02	225	3286	0.37	0.03	199	3622	0.41	0.03	229	3840
HHC-50×7	0.31	0.02	297	8151	0.28	0.02	337	8426	0.34	0.02	306	8615
HHC-80×7	0.31	0.02	320	10367	0.32	0.02	301	12915	0.36	0.02	293	16928
HHC-110×7	0.30	0.01	418	39883	0.33	0.01	451	43686	0.34	0.02	309	44249

TABLE 3.5 – Some extreme points of the Pareto fronts

		f_1^{min}, f_2^{min}			f_2^{min}, f_1^{min}			f_3^{min}, f_1^{min}		
		f_1	f_2	f_3	f_1	f_2	f_3	f_1	f_2	f_3
	Lexicographic									
	HHC-3×7	338.80	72	5.04	338.80	72	5.04	930.74	78	1.01
	HHC-6×7	822.28	118	8.50	838.34	108	8.48	1094.95	127	1.05
	AUGMECON									
	HHC-3×7	338.80	72	5.04	338.80	72	5.04	930.74	78	1.01
	HHC-6×7	822.28	118	8.50	836.55	110	7.98	1094.95	127	1.05
$DS_{0\%}$	ϵ-gLNS-MDLS									
	HHC-3×7	338.80	89	5.04	866.94	72.00	3.85	930.74	92.00	1.01
	HHC-6×7	822.28	232	8.89	882.47	108	7.43	1094.95	127	1.05
	HHC-20×7	864.93	1089	13.62	3023.68	131	9.31	3631.78	718	0.70
	HHC-50×7	2107.16	2368	18.62	6754.38	405	13.71	6000.39	1926	0.71
	HHC-80×7	3629.34	3899	34.04	9095.92	1479	21.73	9964.27	3044	2.16
	HHC-110×7	6361.74	5205	35.94	13944.14	1626	32.15	12581.10	4418	3.14
	Lexicographic									
	HHC-3×7	433.00	72	6.04	433	72	6.04	930.74	78	2.01
	HHC-6×7	822.28	112	10.51	854.04	101	10.10	1042.43	135	2.24
	AUGMECON									
	HHC-3×7	433.00	72	6.04	433.00	72.00	6.04	930.74	78.00	2.01
	HHC-6×7	822.28	112	10.51	955.67	111	4.47	1043.00	130	2.24
$DS_{25\%}$	ϵ-gLNS-MDLS									
	HHC-3×7	433.00	72	6.04	433.00	72.00	6.04	930.74	85.00	2.01
	HHC-6×7	822.28	219	10.51	886.20	102	8.73	1043.00	130	2.24
	HHC-20×7	1054.92	1155	21.25	3137.70	177	12.79	3894.01	822	0.80
	HHC-50×7	2316.32	2294	21.77	6287.99	301	17.03	6424.31	1794	0.99
	HHC-80×7	4806.15	3966	39.30	11667.64	1244	29.19	10981.93	3041	2.69
	HHC-110×7	6427.16	5453	39.66	14094.47	1345	32.02	12565.81	4298	4.01
	Lexicographic									
	HHC-3×7	558.60	72	6.04	558.60	72	6.04	1040.64	78	2.01
	HHC-6×7	822.28	111	11.29	822.85	101	11.29	1158.63	129	2.09
	AUGMECON									
	HHC-3×7	558.60	72	6.04	558.60	72.00	6.04	1040.64	78.00	2.01
$DS_{50\%}$	HHC-6×7	822.28	111	11.29	822.28	111	11.29	1158.63	129	2.09
	ϵ-gLNS-MDLS									
	HHC-3×7	558.60	89	6.04	882.64	72	5.10	1040.64	92	2.01
	HHC-6×7	822.28	235	11.29	822.28	111	11.29	1278.89	269	2.18
	HHC-20×7	1537.67	1091	23.60	3929.68	219	12.92	4213.71	754	1.05
	HHC-50×7	3170.77	2285	23.83	7893.60	386	20.77	7056.83	1882	0.91
	HHC-80×7	5894.99	3808	42.94	14886.04	1228	37.84	12452.28	2879	2.68
	HHC-110×7	9999.27	5143	39.06	17765.62	1126	45.09	15329.39	4711	3.28

When operational costs attain their nadir, a concurrent decrease in both patient and caregiver satisfaction has been observed. When f_1 is minimized first, followed by f_2 reaching its lowest value, the workload imbalance, represented by f_3, increases significantly, by 472.30%. Similarly, if f_2 attains its minimum before f_1, the workload imbalance rises by an average of 356.59%. This data implies that while we can reduce operational cost and time window violation, it severely compromises f_3. Therefore, home health companies must be cautious when minimizing costs related to f_1 and f_2. Such a trade-off emphasizes the criticality of stakeholder engagement in comprehending and aligning their priorities and constraints, which is instrumental in informed decision-making and the development of strategies resonating with the needs and preferences of all involved parties.

3.3.4 Managerial recommendations

To gain a more comprehensive understanding of the problem's characteristics and to provide management recommendations, we further analyze the solutions' details when f_1 and f_2 achieve their minimum values on the Pareto front. We exclude the HHC-3×7 and HHC-6×7 datasets due to their small size, which makes a single caregiver sufficient for management.

In Table 3.6, OV denotes the overtime hours logged by internal caregivers, while CE quantifies the cost incurred for the services of external caregivers. As the number of patients with lower GIR levels increases, we observe a corresponding rise in the minimum value of the first objective.

TABLE 3.6 – Details of minimum the operational costs f_1

		f_1^{min}			Internal			External	
		best	avg	gap	TC_{in}	OV	S_{in}	TC_{ex}	CE
$DS_{0\%}$	HHC-20×7	864.93	866.10	0.13%	64.93	0.00	800	0.00	0.00
	HHC-50×7	2107.16	2131.21	1.14%	158.38	12.87	1600	50.30	285.60
	HHC-80×7	3629.34	3661.45	0.88%	205.31	14.00	2400	159.28	850.75
	HHC-110×7	6361.74	6520.61	2.50%	300.51	110.75	3200	458.33	2292.15
$DS_{25\%}$	HHC-20×7	1054.92	1060.62	0.54%	63.42	4.63	800	32.37	154.50
	HHC-50×7	2316.32	2347.08	1.13%	140.17	30.00	1600	76.65	469.50
	HHC-80×7	4806.15	4833.92	5.78%	197.14	133.13	2400	286.48	1789.40
	HHC-110×7	6427.16	6634.39	3.22%	363.17	81.00	4000	234.89	1748.10
$DS_{50\%}$	HHC-20×7	1537.67	1554.60	1.10%	63.21	1.00	800	71.16	602.30
	HHC-50×7	3170.77	3250.45	2.51%	186.10	17.13	2400	65.79	501.75
	HHC-80×7	5895.00	6004.58	1.88%	271.15	70.50	3200	239.40	2113.95
	HHC-110×7	9999.27	10209.87	2.10%	431.16	176.50	4800	531.86	4059.75

*The number of caregivers, the travel cost and stuff cost are from the best value of f_1^{min}.

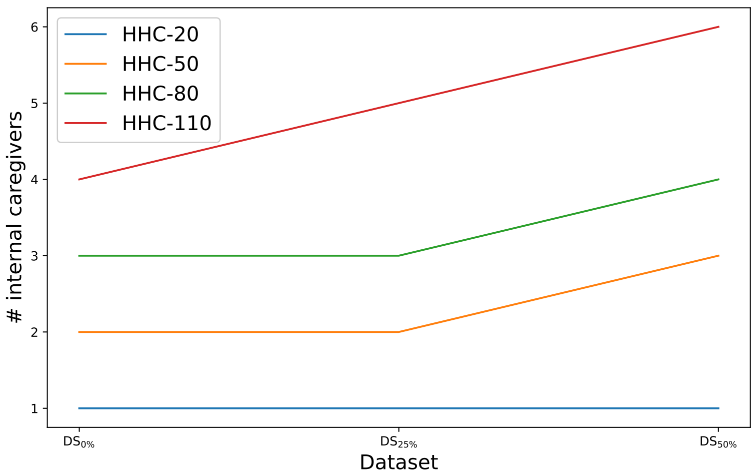


FIGURE 3.3 – The number of internal caregivers.

Table 3.7 shows the service details of internal and external caregivers. Here,

TABLE 3.7 – Details of caregiver workload

		Internal					External					
		N_{in}	TD	ST	V_{Total}	$V_{\text{GIR1\&2}}$	N_{ex}	TD	ST	V_{Total}	$V_{\text{GIR1\&2}}$	ST_{Total}
$\text{DS}_{0\%}$	HHC-20×7	1	85.44	1534	125	0	0	0	0	0	0	1534
	HHC-50×7	2	208.40	3829	264	0	2	32.23	533	19	0	4362
	HHC-80×7	3	270.15	5755	407	0	2	77.07	1579	62	0	7334
	HHC-110×7	4	395.41	7876	494	0	7	280.13	3575	163	0	11451
$\text{DS}_{25\%}$	HHC-20×7	1	83.09	2039	119	34	2	26.49	237	12	1	2276
	HHC-50×7	2	184.44	3875	246	77	3	42.16	692	30	4	4576
	HHC-80×7	3	259.40	6082	370	110	4	175.13	2697	105	20	8779
	HHC-110×7	5	477.85	9578	569	142	4	169.47	2371	89	29	11451
$\text{DS}_{50\%}$	HHC-20×7	1	86.19	2029	99	55	2	4.43	817	30	13	2846
	HHC-50×7	3	244.87	4939	252	143	2	40.81	549	25	9	5488
	HHC-80×7	4	356.77	7924	384	202	4	174.51	2856	91	47	10780
	HHC-110×7	6	567.32	10635	482	243	7	324.26	4887	180	97	15522

*TD: Travel Distance, kilometer; ST: service time, minute; V_{Total}: the total number of visits; $V_{\text{GIR1\&2}}$: the number of visits with GIR level 1 and 2.

N_{in} and N_{ex} represent the number of internal and external caregivers used within a week, respectively. Figure 3.3 illustrates the number of internal caregivers for different instances. As the percentage of high-dependence patients rises, the number of internal caregivers either increases or remains the same. The internal caregivers experience longer travel and service times, as well as more patient visits than external caregivers. Additionally, the values of $V_{\text{GIR1\&2}}$ demonstrate that the internal caregivers handle a higher proportion of visits with patients of high dependence levels than the external ones, indicating that the internal caregivers are responsible for more complex patient care needs.

As the number of patients with high dependence levels (GIR levels 1 and 2) rises, the total workload (ST_{Total}) also increases, necessitating new assignments and scheduling adjustments. To optimize operational costs, it is advisable to allocate more complex tasks to internal caregivers. When there is an uptick in the number of patients with high dependence levels, expanding the team of internal caregivers is a strategic move. Meanwhile, the engagement of external caregivers is influenced by their proximity to the jobs' locations. Therefore, for patients at lower dependence levels (GIR levels 3 and 4), who may incur lower costs and require less intensive care, assigning nearby external caregivers is an efficient strategy. This approach ensures prompt, cost-effective care delivery, adequately meeting the varying needs of patients across different GIR levels.

3.4 Conclusions

In order to address questions Q1 and Q2, two types of caregivers were optimally assigned to the patients, aiming to meet the diverse needs of three principal stakeholders: cost reduction for the manager, high-quality care for the patients, and equitable task allocation for the caregivers. Three objective functions encompassed the minimization of operational costs, penalties associated with caregivers' arrival and departure times exceeding the time windows, and disparities in workload across all caregiver pairs. Continuity of care was incorporated as constraints in this multi-objective optimization problem. Recognizing the computational intensity of the problem, we proposed an ϵ-greedy large neighborhood method embedded within an improved multi-directional local search framework. This approach was used to obtain the Pareto fronts, facilitating the understanding of trade-offs among the three objectives.

Computational experiments were conducted using real-life data. We compared our proposed algorithm with the multi-objective optimization method in existing literature on different instances. The results demonstrated the effectiveness of our approaches, especially in handling large-scale instances. The results of Pareto fronts revealed that reducing operational costs and minimizing time window violations compromised the workload balance. When applying the algorithm to the instances with different patient compositions, we observed that the operational costs rise with higher percentages of patients with high dependence. In making employment decisions, it is recommended that internal caregivers be used to ensure constant and regular work, while external caregivers accommodate fluctuating work. This complementary use of internal and external caregivers is crucial in maintaining high service quality. Meanwhile, our approach provided a tool to fine-tune staffing under different patient compositions.

This chapter focuses on weekly home health care planning. Before the start of a week, managers prepare static schedules for caregivers to follow. However, daily execution often faces uncertain service times of patients, adversely affecting service quality. Thus, robust daily routes and schedules become crucial to accommodate these uncertainties and maintain care quality. Chapter 4 aims to incorporate uncertain service times into daily planning and develop stochastic methods to solve the problem.

Chapter 4

Daily routing and scheduling problems considering uncertainty

Chapter 3 has studied static weekly planning. However, as discussed in Chapters 1 and 2, the uncertain service times can result in delays, reduce service quality, and increase the workload of caregivers. It is important to balance saving travel costs and satisfying both caregivers and patients. Therefore, this chapter aims to address the research question Q3: how to develop robust daily routes and schedules considering uncertain service times as well as the satisfaction of both caregivers and patients. Furthermore, it explores research question Q4, assisting managers in practically choosing an optimal solution from the Pareto front. This chapter is organized as follows: Section 4.1 addresses the model with two objective functions. A stochastic multi-objective optimization method is proposed in Section 4.2. The parameters setting, computational experiments on literature instances, and results analysis are presented in Section 4.3. In Section 4.4, the parameters inherited from the sensitivity analysis are utilized in a real-world application, followed by managerial suggestions. Section 4.5 concludes this chapter.

This chapter regroups several works, including a journal article submitted to the International Journal of Environmental Research and Public Health (currently under review), and two conference papers that have been published in la 23ème édition du congrès annuel de Recherche Opérationnelle et d'Aide à la Décision (ROADEF 2022), and 11e Conférence en Gestion et Ingénierie des Systèmes Hospitaliers (GISEH 2022).

4.1 Problem statement

In this section, a mathematical model and its components representing the extension of VRPTW in HHC have been formulated. From the perspective of graph theory: let $G = (V, A)$, where $V = \{0\} \cup N = \{1, 2 \ldots, n\}$ is the vertex set and represents the depot and the patients. $A = \{(i, j) \in V, i \neq j\}$ is the set of arcs. We aim to find minimum-cost routes that serve the vertices once and satisfy the side constraints of arc R⊆A.

It is assumed that all caregivers leave the depot at time 0. Caregiver $k \in K$ starts from the facility, moves once to each patient, and returns to the depot. Each caregiver has the qualification level Q_k. Fixed levels of qualification $Q = \{1, ..., q\}$ are defined in this paper. A visit is therefore allowed only if the patient's requirement is lower or equal to the qualification level of the caregiver.

Patients are spread across different locations. Each patient i has requirement RC_i for a specific level of the caregiver. Patients' requirement set is aligned with the qualification set of the caregivers. The time window $[e_i, l_i]$ of the patient i is defined as the earliest time of starting service and the latest time of ending service that can be tolerated by the patient. If the caregiver arrives earlier than e_i, the service will start before reaching e_i; If the arrival is after e_i, the service begins immediately. The caregiver will leave immediately after serving for δ_i. The departure time can be calculated as (4.1).

$$d_{ik} = \max(a_{ik}, e_i) + \delta_i \tag{4.1}$$

The arrival time of patient j is

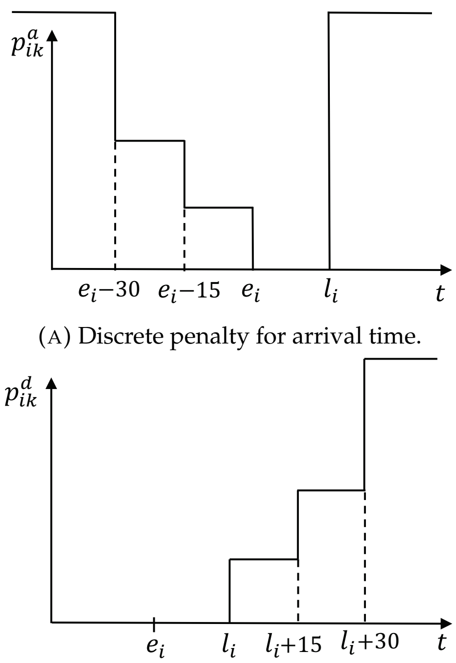
$$a_{jk} = d_{ik} + t_{ij} = \max(a_{ik}, e_i) + \delta_i + t_{ij}. \tag{4.2}$$

Caregivers who arrive too early may face long waiting times, while arriving too late can lead to decreased patient satisfaction. A penalty cost is introduced in objective functions when caregivers arrive or leave outside the time window. Different penalty values are assigned to $[0, e_i - 30]$, $(e_i - 30, e_i - 15]$, $(e_i - 15, e_i]$, $(e_i, l_i]$, $(l_i, l_i + 15]$, $(l_i + 15, l_i + 30]$, $(l_i + 30, \infty]$. In other words, we have two loose time windows $[e_i - 30, l_i + 30]$ and $[e_i - 15, l_i + 15]$ and one tight time window $[e_i, l_i]$. Discrete penalties can prevent a few patients from suffering from large delays. When $y_{ik} = 1$, the penalty is determined using (4.3) and (4.4); otherwise, p^a_{ik} and p^a_{ik} are equal to 0. Fig.

4.1 shows the penalties schematically.

$$p^a_{ik} = \begin{cases} \beta_0, a_{ik} \leq e_i - 30 \\ \beta_1, e_i - 30 < a_{ik} \leq e_i - 15 \\ \beta_2, e_i - 15 < a_{ik} \leq e_i \\ \beta_3, e_i < a_{ik} \leq l_i \\ \beta_4, l_i < a_{ik} \end{cases}, \tag{4.3}$$

$$p^d_{ik} = \begin{cases} \alpha_0, d_{ik} \leq l_i \\ \alpha_1, l_i < d_{ik} \leq l_i + 15 \\ \alpha_2, l_i + 15 < d_{ik} \leq l_i + 30 \\ \alpha_3, l_i + 30 < d_{ik} \end{cases} \tag{4.4}$$



(A) Discrete penalty for arrival time.

(B) Discrete penalty for departure time.

FIGURE 4.1 – Illustration of discrete penalty.

The notation to describe the model is summarized in Table 4.1.

TABLE 4.1 – Notations.

Notation	Definition
Sets	
N	set of patients
V	set of depot and patients
A	all arcs
K	set of caregivers
Q	set of levels of qualification
RC	set of requirements of patients for levels of caregivers
H	set of number of intervals divided by departure time
G	set of number of intervals divided by arrival time
Parameters	
i,j	index of patients
k	index of caregivers
c_{ij}	travel cost between i and j
t_{ij}	travel time between i and j, is c_{ij}
Q_k	level of qualification of caregiver k
RC_i	requirement of patient i for qualification level of a caregiver
δ_i	service time of patient i
e_i, l_i	time window of patient i
m,n	minimal number of patients and maximal number of patients that one caregiver is able to visit
α_h	degree coefficient if departure time is located at h^{th} interval
β_g	degree coefficient if arrival time is located at g^{th} interval
M	a positive large number
Decision variables	
x_{ijk}	binary decision variable, 1 if caregiver k moves from i to j, 0 otherwise

Notation	Definition
y_{ik}	binary decision variable, 1 if patient i is served by caregiver k, 0 otherwise
a_{ik}	arrival time of caregiver k's visit to patient i
d_{ik}	departure time that caregiver k leaves patient i
p^d_{ik}, p^a_{ik}	continuous decision variable, the penalty that arrival time and departure time are outside of time windows
w^d_{ik}, w^a_{ik}	auxiliary variables, continuous, $p_{ik} = w_{ik} * y_{ik}, \forall i \in N, \forall k \in K$
u_{ig}, v_{ih}	binary decision variable, 1 if caregivers' arrival (departure) time at patient i is located at g^{th} (h^{th}) interval, 0 otherwise
r_i	binary decision variable, 1 if caregiver arrives after e_i

The objective functions and constraints are formulated by (4.5)-(4.38).

$$\min \sum_{i,j \in A} \sum_{k \in K} c_{ij} x_{ijk} \tag{4.5}$$

$$\min \sum_{i \in N} \sum_{k \in K} P_{ik} \tag{4.6}$$

$$P_{ik} = p^d_{ik} + p^a_{ik} \tag{4.7}$$

$$s.t. \sum_{j \in V, i \neq j} \sum_{k \in K} x_{ijk} = 1, \forall i \in N \tag{4.8}$$

$$\sum_{i \in V, i \neq j} x_{ijk} = \sum_{i \in V, i \neq j} x_{jik}, \forall k \in K, \forall j \in V \tag{4.9}$$

$$\sum_{j \in N} x_{0jk} = 1, \forall k \in K \tag{4.10}$$

$$y_{ik} * RC_i \leq Q_k, \forall i \in N, \forall k \in K, \forall Q_k \in Q \tag{4.11}$$

$$y_{ik} = \sum_{j \in V, i \neq j} x_{ijk}, \forall i \in N, \forall k \in K \tag{4.12}$$

$$m \leq \sum_{i \in N} \sum_{j \in V} x_{ijk} \leq n, \forall k \in K \tag{4.13}$$

$$d_{ik} + t_{ij} \leq a_{jk} + (1 - x_{ijk}) * M, \forall i \in N, \forall j \in V, \forall k \in K, i \neq j \tag{4.14}$$

$$d_{ik} + t_{ij} \geq a_{jk} - (1 - x_{ijk}) * M, \forall i \in N, \forall j \in V, \forall k \in K, i \neq j \tag{4.15}$$

$$a_{jk} \leq t_{0j} + (1 - x_{0jk}) * M, \forall j \in N, \forall k \in K \tag{4.16}$$

$$a_{jk} \geq t_{0j} - (1 - x_{0jk}) * M, \forall j \in N, \forall k \in K \tag{4.17}$$

$$d_{0k} = 0, \forall k \in K \tag{4.18}$$

$$d_{ik} \geq a_{ik} + \delta_i, \forall i \in N, \forall k \in K \tag{4.19}$$

$$d_{ik} \geq e_i + \delta_i, \forall i \in N, \forall k \in K \tag{4.20}$$

$$d_{ik} \leq a_{ik} + \delta_i + (1 - r_i) * M, \forall i \in N, \forall k \in K \tag{4.21}$$

$$d_{ik} \leq e_i + \delta_i + r_i * M, \forall i \in N, \forall k \in K \tag{4.22}$$

$$\sum_{h \in H} v_{ih} = 1, \forall i \in N \tag{4.23}$$

$$d_{ik} \leq l_i * v_{i0} + (l_i + 15) * v_{i1} + (l_i + 30) * v_{i2} + M * v_{i3}, \forall i \in N, \forall k \in K \tag{4.24}$$

$$d_{ik} \geq l_i * v_{i1} + (l_i + 15) * v_{i2} + (l_i + 30) * v_{i3}, \forall i \in N, \forall k \in K \tag{4.25}$$

$$w_{ik}^d = \sum_{h \in H} \alpha_h v_{ih}, \forall i \in N, \forall k \in K \tag{4.26}$$

$$p_{ik}^d \leq w_{ik}^d + M * (1 - y_{ik}), \forall i \in N, \forall k \in K \tag{4.27}$$

$$p_{ik}^d \geq w_{ik}^d - M * (1 - y_{ik}), \forall i \in N, \forall k \in K \tag{4.28}$$

$$p_{ik}^d \leq M * y_{ik}, \forall i \in N, \forall k \in K \tag{4.29}$$

$$\sum_{g \in G} u_{ih} = 1, \forall i \in N \tag{4.30}$$

$$a_{ik} \leq (e_i - 30) * u_{i0} + (e_i - 15) * u_{i1} + e_i * u_{i2} + l_i * u_{i3} + M * u_{i4}, \forall i \in N, \forall k \in K \tag{4.31}$$

$$a_{ik} \geq (e_i - 30) * u_{i1} + (e_i - 15) * u_{i2} + e_i * u_{i3} + l_i * u_{i4}, \forall i \in N, \forall k \in K \tag{4.32}$$

$$w_{ik}^a = \sum_{g \in G} \beta_g u_{ig}, \forall i \in N, \forall k \in K \tag{4.33}$$

$$p_{ik}^a \leq w_{ik}^a + M * (1 - y_{ik}), \forall i \in N, \forall k \in K \tag{4.34}$$

$$p_{ik}^a \geq w_{ik}^a - M * (1 - y_{ik}), \forall i \in N, \forall k \in K \tag{4.35}$$

$$p_{ik}^a \leq M * y_{ik}, \forall i \in N, \forall k \in K \tag{4.36}$$

$$x_{ijk}, y_{ik}, r_i, u_{ig}, v_{ih} \in \{0,1\}, \forall i \in V, \forall j \in V, \forall k \in K, \forall g \in G, \forall h \in H, i \neq j \tag{4.37}$$

$$a_{ik}, d_{ik}, p^d_{ik}, p^a_{ik}, w^d_{ik}, w^a_{ik} \geq 0, \forall i \in N, \forall k \in K \tag{4.38}$$

The first objective function (4.5) is to minimize the travel cost. The second objective function (4.6) represents the penalty cost to be minimized. A smaller penalty cost indicates greater satisfaction for both caregivers and patients. Constraints (4.8) ensure that a caregiver is assigned to exactly one route. Constraints (4.9) mean each caregiver visits the patient and then leaves the patient. Constraints (4.10) indicate that caregivers start from the depot and return to the depot after finishing services. Caregivers can perform the service only if their qualification levels are satisfied by constraints (4.11)-(4.12). Constraints (4.13) indicate that each caregiver must serve a certain number of patients in relation to the workload balance. Constraints (4.14)-(4.22) guarantee the schedule feasibility and make subtours impossible. Note that, a_{ik} and d_{ik} are meaningless whenever patient i is not visited by caregiver k. (4.19)-(4.22) aim to convert the (4.1) into linear. (4.23)-(4.36) are the variants of (4.4) and (4.3). Constraints (4.37)-(4.38) set the domains of decision variables.

For the MILP model, the optimal solutions of each objective function can be obtained by Gurobi Solver. We use a weighted sum method to get the approximation of a Pareto optimal set. The sum of the weights of two objective satisfies $\omega_1 + \omega_2 = 1$ but in the next iteration, $\omega_1^{(t+1)} = \omega_1^{(t)} + \Delta$ and $\omega_2^{(t+1)} = \omega_2^{(t)} - \Delta$. We keep only non-dominated solutions from T solutions which are obtained after T iterations.

4.2 Multi-objective algorithms

Our proposed method ALNS-MDLS is divided into two main components: the enhanced Multi-directional Local Search (MDLS) described in Section 4.2.1, and the Adaptive Large Neighborhood Search (ALNS) detailed in Section 4.2.2. To address uncertain service times, the stochastic version of ALNS-MDLS is proposed in Section 4.2.3.

4.2.1 Enhanced multi-directional local search algorithm

The Multi-Directional Local Search method was first proposed by Tricoire Tricoire (2012). Each local search is performed for a single objective (direction) iteratively to improve the non-dominated front F. Each local search works separately without considering the importance of the objectives. Only non-dominated solutions are kept after one iteration. This strategy has fewer parameters and can yield well-spread solutions. The savings algorithm is a kind of constructive heuristic and can be used to construct the initial solution. In each direction, we use the ALNS to improve solutions and put the solutions in F. The Deb non-dominated sorting method is used to keep the non-dominated front after each iteration. F is saved as an ordered list to reduce the number of times executing non-dominated sorting.

Our enhanced MDLS differentiates from the original algorithm in two ways. Firstly, unlike the original algorithm where a single solution from set F initiates the next iteration, our approach retains multiple solutions in each direction to enhance diversity. Secondly, our method selects solutions from F based on crowding distance, drawing inspiration from NSGA-II (Deb et al., 2002), rather than making random selections. The crowding distance of point i in F can be regarded as the perimeter of the hypercube which is surrounded by the two adjacent points $i-1$ and $i+1$. The two boundary points are assigned to a very large number. The solution with a larger crowding distance is more likely to be chosen.

4.2.2 Adaptive large neighborhood search (ALNS)

In the ALNS, various destroy and repair operators are selected adaptively to construct new solutions, which are accepted if their objective function values meet the record-to-record criterion, as detailed as follows.

Destroy and repair operators

Three destroy operators and three repair operators are designed based on the previous work in Ropke and Pisinger (2006). We remove nodes from the solution by the destroy operators and then insert the removed nodes by the repair operators. To satisfy the constraints (4.13), we select the routes with over m patients for destruction and those with less than n patients for repair. We choose the routes where caregivers meet patients' demands to respect the constraints (4.11).

A certain number of nodes are randomly removed and inserted by the random destroy operator and the random repair operator respectively. These operators can easily be implemented to run faster than others. The worst destroy operator chooses the nodes with the largest saving that appear to be placed in the wrong position in the solution, while the relatedness destroy operator tends to select the nodes that are similar and can easily be exchanged. The relatedness of the first objective function (4.5) can be calculated by $\frac{1}{c_{ij}/c_{\max}+v}$, while the objective function (4.6) by $\frac{1}{(|e_i-e_j|+|l_i-l_j|)/tw_{\max}+v}$, where $c_{\max}$ denotes the largest cost of all pairs of i and j, $tw_{\max}$ is length of the longest time window. If the node i and the node j are in the same route, $v = 0$; otherwise, $v = 1$. We iteratively find the node with minimum cost position in the greedy repair operator. But for the nodes that are expensive to insert in the last iteration, there are not many opportunities for inserting them because many of the routes are "full". The regret operator chooses the nodes from the removal set by calculating $i = \arg\max_{i\in u}[\sum_{j=1}^{k}(\Delta f_i^j - \Delta f_i^0)]$, where u is the removal set, and Δf_i^j denotes the insertion value of the node i in the j^{th} cheapest insertion position. This method selects the insertion that has a larger possibility to improve the overall performance than the greedy method. Appendix A contains the details of the destroy operators and the repair operators.

Adaptive weight adjustment and acceptance criterion

Only one destroy operator and one repair operator are chosen by probability $\frac{w_j}{\sum_{i=1}^{k} w_i}$ in one iteration, where w_j is the weight of the j^{th} operator to be chosen, $i \in \{1, 2, ..., k\}$. The entire search is divided into several segments. A segment is a number of iterations of the ALNS. The weight w_j is automatically updated after a segment and is calculated by the formula (4.39).

$$w_j = (1-\gamma) * w_i + \gamma * \frac{r^{\text{score}}}{o^{\text{num}}} \tag{4.39}$$

The variable o^{num} means the usage frequency of operator i in the latest segment. The reaction factor γ controls how quickly the weight adjustment algorithm reacts to the changes in the effectiveness of the heuristics. The r^{score} can take three values: r_1, r_2, and r_3, corresponding to three types of acceptance criteria, which assess the heuristic's recent performance. A high score corresponds to a better performance. More specifically, in the record-to-record method, a neighborhood solution generated by the destroy and repair operators is always accepted if it outperforms the current solution, the best solution, and the sum of the best solution and deviation.

We allow infeasible solutions where some patients in the removal set may not be scheduled by a repair operator. In this case, the number of remaining patients incurs penalties in two objectives by the following formulas:

$$\min \sum_{i,j \in V} \sum_{k \in K} c_{ij} x_{ijk} + \eta \sum_{i \in N} z_i, \tag{4.40}$$

$$\min \sum_{i \in N} \sum_{k \in K} P_{ik} + \eta \sum_{i \in N} z_i, \tag{4.41}$$

where $z_i = 1$ if the patient i can not be inserted; otherwise, $z_i = 0$.

Fig. 4.2 and Algorithm 7 summarize the ALNS-MDLS.

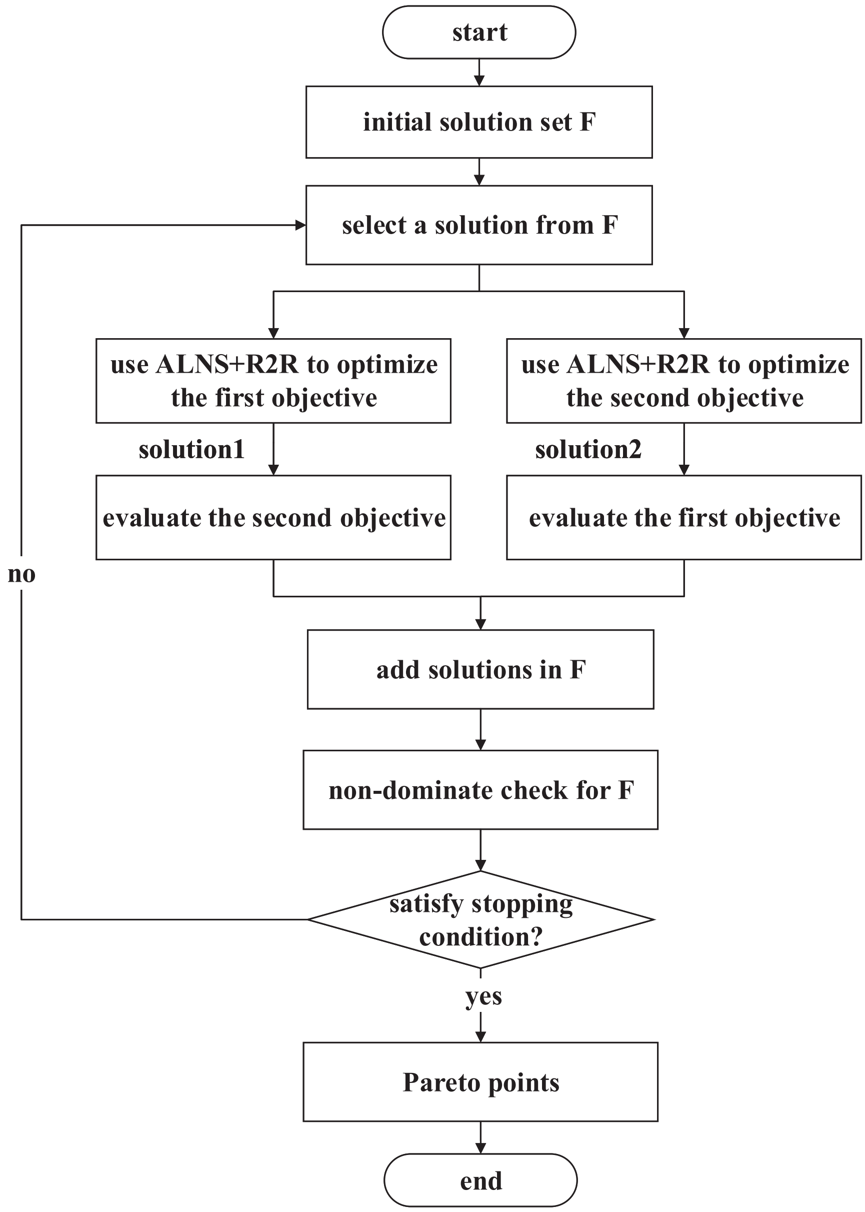


FIGURE 4.2 – Structure of ALNS-MDLS.

Algorithm 7 ALNS-MDLS

Input: a set F only including an initial solution x, repair operators, destroy operators, deviation d, $iter^{\mathrm{seg}}$, r_1, r_2, r_3

Output: the Pareto front F

```
repeat
  x^cur ← crowd_distance(F)
  x^best ← x^cur
  G ← ∅
  for k ←1 to K do
    for j ←1 to iter^ALNS do
      for i ←1 to iter^seg do
        choose a destroy operator destroy_i and a repair operator repair_i by weight w
        x^new ← repair_i(destroy_i(x^cur))
        if f(x^new) < f(x^cur) then
          x^cur ← x^new
          if f(x^new) < f(x^best) then
            x^best ← x^new
            update score by r_1
          else
            update score by r_2
          end if
        else if f(x^new) < (1 + d)f(x^best) then
          x^cur ← x^new
          update score by r_3
        end if
      end for
      update w by formula (4.39)
    end for
    add solutions of k^th objective to G
  end for
  F ← Deb_nondominated_sorting(F, G)
until stopping criterion is met
```

4.2.3 Stochastic method

An objective function of a stochastic optimization problem can be written as $f(x, Y(\omega))$, where x is the decision variable, and Y is a random variable that associates a real number to each element ω of a sample space Ω. We simply write it as $f(x, \omega)$. Without loss of generality, we consider the form of the stochastic $\mathcal{K}$-objective optimization problem as Gutjahr and Pichler, 2016:

$$\min(f_1(x, \omega), f_2(x, \omega), ..., f_{\mathcal{K}}(x, \omega)) \quad s.t. x \in \mathcal{X}. \tag{4.42}$$

It can be reduced to a deterministic model by defining f_i as an s-dimensional vector $\mathcal{F}_i^{(s)}(f_i(x, \omega))$ (for other transformations see Abdelaziz, 2012 and Campi and Garatti, 2018). Expectation $\mathbb{E}(f_i(x, \omega))$, which is one of specific functional $\mathcal{F}_i^{(s)}$, is used in this paper. To make the expectation computationally tractable, each objective function is estimated by a sample average approximation method. $\mathbb{E}(f_i(x, \omega))$ can be replaced by an unbiased consistent estimator (Gutjahr, 2005).

$$\frac{1}{U} \sum_{\omega_v \in S} f_i(x, \omega_v), \tag{4.43}$$

where $S = \{\omega_1, \omega_2, ..., \omega_U\}$ is a fixed finite set of scenarios drawn in advance.

We assume the service time follows a certain probability distribution which is known in advance. We generate U scenarios from the distribution for each patient by means of the Monte Carlo method. The second objective function varies under different scenarios. The first objective function is not calculated by the service time, but it is indirectly affected.

The objective function (4.6) can be rewritten as:

$$\tilde{f}_2 = \frac{1}{U} \sum_{s \in S} \sum_{R_k \in R} \sum_{r_j \in R_k} P_{r_j}^s, \tag{4.44}$$

where $P_{r_j}^s$ denotes the penalty of node r_j under scenario s; $R = \{R_1, R_2 ..., R_{|K|}\}$ is a set of routes. Here, $|K|$ represents the number of routes, and R_k means the k^{th} route in the set R. In the stochastic version of ALNS-MDLS, the second objective function f_2 is replaced by $\tilde{f}_2$, and it can be calculated by Algorithm 8 under different scenarios of service time.

Algorithm 8 Stochastic simulation for computing the expectation of penalty cost

Input: a solution, $s = 1$, $sum = 0$

Output: estimate expected value of penalty cost

while $s \leq U$ **do**
  for R_k in R **do**
    for r_j in R_k **do**
      generate $\widetilde{\delta}_{r_j}^{(t)}$ from sample space according to the probability measure.
      compute the arrival time and departure time according to (4.1) and (4.2), then calculate P_{r_j} by (4.4) and (4.3).
      $sum = sum + P_{r_j}^{s}$
    end for
  end for
end while
the estimated expected value is $sum = sum/U$

4.3 Computational study

In this section, our experiment datasets and settings are presented. Some metrics are used to evaluate the quality of non-dominated solutions and the efficiency of multi-objective algorithms. We analyze the results obtained by an exact solution approach (the Gurobi Solver) and two approximate solution approaches (the ALNS-MDLS and the stochastic ALNS-MDLS). For all the experiments we have used an Intel(R) Core (TM) i5-10310U CPU (@ 2.21 GHz) CPU with 16GB of RAM memory.

The computational study was carried out on two types of data sets. First, and to extend the study of the performance and robustness of the proposed method, a series of tests was created based on literature instances. Second, a several dataset is taken directly from the field. It is used to analyze the proposed solutions in terms of business practices and to draw out managerial insights which is presented in Section 4.4.

4.3.1 Data sets and experimental setup

No benchmark results exist in the literature for our problem. Hence, we generate six different types of (C1, C2, R1, R2, RC1, RC2) instances based on the Solomon data set (Solomon, 1987) and compare the metrics to test whether the algorithm fits for the instances with different sizes and characteristics. The data sets that support the findings of this study are openly available in "figshare" at `http://doi.org/10.6084/m9.figshare.21339072`. We have compared problems involving 25, 50, and 100 patients, focusing on the characteristics including the geographical data, the length of scheduling horizon and the proportion of time-constrained patients. The geographical data is randomly generated in R1 and R2, clustered in C1 and C2, and a mix of both structures in RC1 and RC2. The sets R1, C1 and RC1 have a short scheduling horizon while the sets R2, C2 and RC2 have a longer one. Each type comprises 4 sets. For example, C1 consists of C1-a, C1-b, C1-c and C1-d. The only distinction between C1-a and the other three sets lies in the presence of some patients having time windows that are scarcely constrained in sets C1-b, C1-c, and C1-d.

Each patient is available only between the ready time and the due time. Distance is Euclidean, and the value of travel time is equal to the value of distance between two nodes. We assign random values to patient requirements for caregiver levels and service times. To be more practical, we diversity the service time of each patient. The mean value of service time δ accounts for twenty to sixty percent of the time window. We assume that the service time of each patient is an independently normally distributed random variable and follows $N(\mu_i, \sigma_i^2)$. We set $\mu = \delta$ and $\sigma = \sqrt{\frac{\delta}{5}}$. We assume that 3 caregivers are assigned to 25 patients, 5 caregivers to 50 patients, and 10 caregivers to 100 patients.

Before starting the problem-solving, the parameters are set. For solving the MILP by Gurobi Solver, we set Δ as $1/50$. The ALNS is affected by random factors, so we utilize the average value of the metrics from five runs for every experiment. In each iteration, the number of nodes removed by destroy operators is randomly set between 2 and 4, as reflected in the variable q in Algorithms 9 and 10 (refer to **??**). η in formula (4.40) is set to 1000. Given the impracticability of testing all hyperparameter combinations, we employ Bayesian optimization for efficient exploration (Snoek, Larochelle, and Adams, 2012). It is assumed the hyperparameters are in a black box (an unknown function), with the function's output evaluated via a metric known as the Hypervolume indicator (detailed in Section 5.2). In Bayesian optimization, it assumes the unknown function stems from a Gaussian process prior, updating the posterior distribution with new observations. An acquisition function is chosen for the next evaluation point. The tuned hyperparameters of the proposed method and their best values after 50 iterations are shown in Table 4.2.

TABLE 4.2 – Hyper parameters.

Notation	Definition	Value
r_1	score if $f(x^{\text{new}}) < f(x^{\text{cur}})$	21.38
r_2	score if $f(x^{\text{new}}) < f(x^{\text{best}})$	18.93
r_3	score if $f(x^{\text{new}}) < (1+d)f(x^{\text{best}})$	7.08
γ	coefficient of weight (see formula (4.39))	0.68
$iter^{\text{seg}}$	the number of iterations to update weight	4
$iter^{\text{ALNS}}$	the number of iterations of ANLS of each direction	19
d	percentage of the objective value of the best solution	0.13

4.3.2 Performance metrics

The metrics are used to measure convergence and diversity (diversity includes ductility and uniformity) of the solutions (Riquelme, Von Lücken, and Baran, 2015), including the number of Pareto optimal points (N), the Hypervolume indicator (HV) and the Spread metric (S) (Audet et al., 2021).

The objective values are scaled between 0 and 1 before calculating these metrics. HV gives the volume x enclosed by a reference point and the solutions and is shown in the following formula (4.45):

$$HV = \bigcup\nolimits_{x \in A} V(x, R). \tag{4.45}$$

The reference point R is commonly set to the point (1,1) for minimizing a bi-objective problem. This metric measures the convergence and the diversity. A larger value of hypervolume signifies better quality of the solutions. The spread metric evaluates the diversity of solutions and is given by:

$$sp = \frac{d_f + d_l + \sum_i^{n-1} |d_i - \overline{d}|}{d_f + d_l + \overline{d}(n-1)}, \tag{4.46}$$

where d_f and d_l are the Euclidean distances between the extreme solutions in true

Pareto front and non-dominated solutions (NDS). $\overline{d}$ is the average of the whole distance d_i, and d_i is the Euclidean distance between one solution and the next nearest solution, where $i \in [1, |NDS| - 1]$.

We assume the extreme values of a true Pareto front are (0,1) and (1,0). Smaller values indicate better distribution.

4.3.3 Deterministic bi-objective solutions

This is a base case in which the service times are considered as deterministic quantities. We compare the deterministic ALNS-MDLS and the Gurobi Solver to measure the performance of our proposed method.

The two extreme points of Pareto front ($[f_1^{\min}, f_2]$ and $[f_2^{\min}, f_1]$), HV, S, the CPU execution time of the program measured by seconds (TCPU) and the number of Pareto points (N) for the small size (10 patients) and the real-life size (25, 50 and 100 patients) instances are shown in Table 4.3. We find that the running time of the Gurobi Solver is much longer than that of the ALNS-MDLS. The extreme values of the two objectives are very close. The outcomes produced by the proposed method, which encompass a broad-range Pareto front, are remarkably comparable to those of the Gurobi Solver while requiring less time. Moreover, the results of the ALNS-MDLS involve more solutions than the Gurobi Solver. Real-world instances typically contain a larger number of patients. With large instances solving the problem to optimality is troublesome, and we found no solution within a limited time by Gurobi Solver. We, therefore, considered the ALNS-MDLS. Computation times shown in Table 4.3 indicate that the ALNS-MDLS is effective. Other metrics show the Pareto fronts achieved by the proposed method are well distributed and have satisfactory diversity.

TABLE 4.3 – Results of Gurobi Solver and ALNS-MDLS.

	min f_1		min f_2					
	f_1	f_2	f_2	f_1	HV	S	N	TCPU
Gurobi								
10-C1	128.64	52.00	32.00	147.69	0.72	0.15	4.00	293.13
10-C2	163.35	62.00	39.00	189.59	0.61	0.09	4.00	625.47
10-R1	194.47	85.00	21.00	258.70	0.68	0.07	6.00	233.86
10-R2	194.47	68.00	18.00	344.84	0.66	0.06	7.00	271.53
10-RC1	218.06	81.00	23.00	336.60	0.74	0.12	8.00	285.31
10-RC2	230.23	75.00	18.00	462.66	0.68	0.02	10.00	293.19
D_MDLS								
10-C1	128.64	52.00	33.00	153.64	0.74	0.15	4.00	21.35
10-C2	163.35	62.00	39.00	189.59	0.62	0.07	6.00	20.55
10-R1	194.47	85.00	21.00	258.70	0.69	0.12	8.00	23.15
10-R2	194.47	68.00	18.00	344.84	0.68	0.00	18.00	26.70
10-RC1	218.06	81.00	23.00	336.60	0.75	0.04	16.00	31.90
10-RC2	230.23	75.00	18.00	462.66	0.71	0.004	15.00	26.11
%	**0.00**	**0.00**	0.005	0.007	2.45	-29.02	65.08	-91.60
D_MDLS								
25-C1	182.33	108.00	12.50	527.05	0.76	0.06	25.75	64.08
25-C2	240.45	131.75	26.00	497.09	0.86	0.04	19.00	59.19
25-R1	352.94	162.00	48.75	500.59	0.66	0.05	19.00	60.40
25-R2	350.88	166.50	7.00	820.81	0.80	0.03	24.25	63.59
25-RC1	294.99	135.50	5.25	386.91	0.71	0.04	26.75	58.32
25-RC2	294.99	149.00	3.75	935.51	0.78	0.04	28.25	61.94
50-C1	344.90	226.25	20.50	1220.02	0.72	0.02	39.50	138.76
50-C2	447.31	176.00	27.50	1476.68	0.72	0.02	34.00	134.35
50-R1	564.50	360.25	139.25	947.44	0.77	0.06	29.50	136.68
50-R2	570.02	251.00	2.50	1419.08	0.73	0.02	36.00	139.71
50-RC1	529.73	235.00	26.00	665.48	0.63	0.04	29.50	144.86
50-RC2	591.95	253.75	1.75	1721.49	0.73	0.03	37.50	145.97
100-C1	823.09	414.25	38.75	3297.75	0.68	0.01	56.25	256.97
100-C2	887.19	501.25	41.75	3417.91	0.74	0.01	54.00	241.71
100-R1	935.09	621.75	141.00	1526.37	0.70	0.01	44.25	249.72
100-R2	941.73	530.50	14.25	2692.53	0.72	0.02	55.25	256.78
100-RC1	1006.43	613.50	108.25	1610.18	0.70	0.02	42.00	249.80
100-RC2	1019.04	468.50	8.50	3326.02	0.74	0.03	48.75	256.62

The numerical simulation processes of stochastic programming require a lot of time. Utilizing the Gurobi Solver to derive solutions for the stochastic model would require significantly more time compared to the deterministic model. The proposed method is adequate to find satisfactory solutions. The performances of the proposed method are very close to the best-found solutions obtained by the Gurobi Solver. Therefore, for large instances, we propose employing the stochastic version of ALNS-MDLS to solve the problem under uncertainty.

4.3.4 Stochastic bi-objective solutions

Table 4.4 shows the metrics and extreme objective function values of the Pareto front of the Stochastic ALNS-MDLS (S_MDLS).

TABLE 4.4 – Results of S_MDLS.

	min f_1		min f_2					
S_MDLS	f_1	f_2	f_2	f_1	HV	S	N	TCPU
25-C1	182.35	110.05	22.73	569.10	0.82	0.06	37.25	1318.99
25-C2	239.94	131.46	28.49	597.37	0.85	0.02	27.25	1256.51
25-R1	349.69	179.25	49.63	516.97	0.75	0.02	20.75	1176.40
25-R2	352.78	121.94	13.03	923.21	0.76	0.02	39.75	1306.94
25-RC1	294.99	114.43	9.89	390.71	0.67	0.01	37.25	1196.68
25-RC2	294.99	140.60	6.24	1129.41	0.82	0.02	41.25	1357.98
50-C1	347.27	219.11	39.74	1375.92	0.80	0.02	47.50	2985.87
50-C2	449.06	199.25	33.66	1618.00	0.78	0.02	47.50	2766.20
50-R1	568.73	298.00	143.21	1020.79	0.81	0.05	26.00	3034.78
50-R2	566.30	248.64	15.29	1624.68	0.75	0.02	49.00	2954.02
50-RC1	539.62	222.38	40.75	737.78	0.74	0.04	31.50	2819.57
50-RC2	570.14	231.47	5.62	2235.57	0.76	0.02	48.25	2991.65
100-C1	831.97	485.78	73.61	3591.33	0.74	0.01	63.75	6370.87
100-C2	916.35	459.38	65.52	3754.04	0.77	0.01	61.75	5635.48
100-R1	948.95	599.82	157.89	1758.63	0.69	0.01	40.25	5790.88
100-R2	947.37	539.87	42.95	2942.96	0.75	0.01	64.00	6022.64
100-RC1	1013.65	559.13	122.02	1727.37	0.70	0.02	45.00	5824.40
100-RC2	1016.62	463.38	19.54	3973.91	0.75	0.01	68.00	6000.49

On average, a considerable difference exists between the solution for minimum travel cost and the solution for minimum penalty cost in terms of both objectives. Hence, the quality of services that decision-makers decide to offer to patients has a significant impact on operating costs, underscoring the need for careful decision-making. The instances C1 and C2 which are clustered have smaller minimum travel costs than R and RC. We compared the results under different lengths of time windows. Data types 2 including C2, R2 and RC2 have longer time windows than type 1. Table 4.4 shows that most of the values of the minimum penalty of type 2 are less than those of type 1.

The Pareto fronts are shown in Fig. 4.3. The data sets 25-C1-a, 25-C1-b, 25-C1-c and 25-C1-d are used to test the effect caused by different percentages of patients with time windows. In 25-C1-a, all patients are available only at certain periods of the day. 28%, 52% and 68% patients are available for the full working time of caregivers for 25-C1-b, 25-C1-c and 25-C1-d respectively. Most patients do not have time windows in the 25-C1-d set. The solutions of 25-C1-d dominate 25-C1-a, 25-C1-b and 25-C1-c.

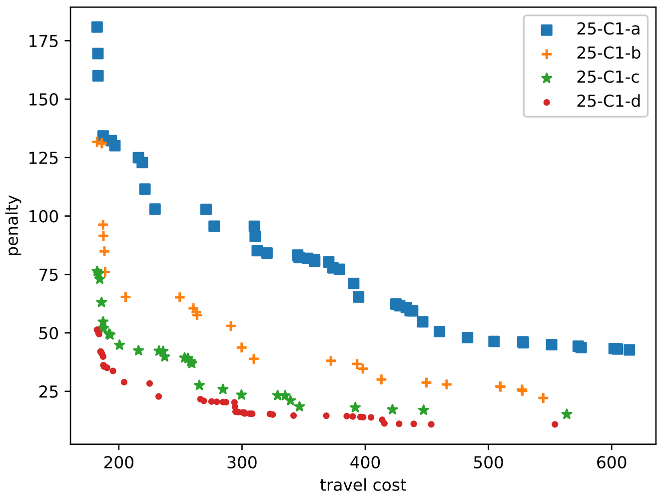


FIGURE 4.3 – Time windows comparison of 25 patients.

We use Algorithm 8 to evaluate the solutions obtained by the Deterministic (original) ALNS-MDLS (D*_MDLS). The number of solutions, HV and S metrics are compared in Table 4.5.

TABLE 4.5 – Deterministic model tested on uncertain environment.

	min f_1		min f_2				
D*_MDLS	f_1	f_2	f_2	f_1	HV	S	N
25-C1	182.33	107.10	26.62	476.12	0.77	0.04	21.50
25-C2	240.45	126.21	30.70	542.94	0.87	0.08	18.50
25-R1	352.94	142.54	52.70	480.16	0.61	0.03	17.50
25-R2	350.88	150.13	16.83	826.24	0.79	0.01	24.00
25-RC1	294.99	122.17	10.97	386.25	0.68	0.03	24.25
25-RC2	294.99	139.35	10.41	947.57	0.78	0.05	30.00
50-C1	344.90	223.08	50.22	1141.53	0.74	0.01	35.00
50-C2	447.32	175.01	41.00	1452.68	0.74	0.09	30.50
50-R1	564.50	329.10	151.81	961.09	0.80	0.09	28.75
50-R2	570.02	232.62	25.47	1423.64	0.74	0.06	30.25
50-RC1	529.73	220.87	49.27	653.99	0.62	0.02	24.00
50-RC2	591.95	244.33	12.24	1814.66	0.74	0.02	35.75
100-C1	823.09	412.46	96.72	3250.29	0.71	0.10	42.25
100-C2	887.20	493.36	73.05	3258.56	0.74	0.01	50.50
100-R1	935.09	579.99	173.21	1544.58	0.68	0.03	37.00
100-R2	941.73	501.50	65.53	2739.26	0.73	0.03	51.50
100-RC1	1006.43	567.45	144.81	1623.38	0.68	0.03	35.50
100-RC2	1019.04	458.41	35.52	3588.20	0.76	0.01	46.00
ANOVA							
F	0.00	0.01	0.37	0.26	2.51	6.08	8.64
p	0.98	0.92	0.55	0.61	0.12	0.02	0.01

It can be inferred from the results that modeling of uncertain service times will increase computing time because the S_MDLS needs to handle more information than only one scenario. Using the proposed stochastic framework more scenarios contribute to the output. The S_MDLS is therefore more realistic. In Table 4.5, the results of the ANOVA test show there is no significant difference of the means of HV and S between the solutions of D*_MDLS and S_MDLS since the P-values are greater than 0.005. This means that the stochastic method can yield solutions that are at least as good as the deterministic one. The proposed stochastic approach is designed to optimize the expected values of objective functions. Its solutions may not be global optimal solutions for the individual scenario but they are robust, providing possible realizations despite uncertain service times.

4.4 Managerial recommendations

4.4.1 The influence of uncertainty on cost and care quality

To identify the behavior of the proposed model and method and examine the influence of uncertain service times on objective values, several sensitivity analysis are performed on the main parameters. To this regard, a small test problem of 25 patients and 3 caregivers is selected. The parameters include the ending time of the loose time windows (ET) and variance of distribution (VD) which can indicate the range of uncertain service times. Each parameter has three levels, namely small, medium and large. To validate the robustness of solutions, we also compare the results of D*_MDLS and S_MDLS. We use the solutions obtained by the deterministic model to evaluate their sensibilities under uncertain service times. We normalize the Pareto set to [0,1] and calculate the distance between the origin and each point in the Pareto set. The solution with a minimum distance (D) is defined as a trade-off solution in our case. The objective values (travel cost (TC), penalty (P), normalized travel cost (NTC), normalized penalty (NP)) of trade-off solutions and minimum distances are summarized in Table 4.6 and 4.7.

l_i means the latest time of the tight time window. If the departure time somehow exceeds the time window by a certain level, there will be a penalty cost. If the departure time lies within $(e_i, l_i]$, $(l_i, l_i + c_1]$, $(l_i + c_1, l_i + c_2]$ or $(l_i + c_2, \infty]$ $(c_1 < c_2)$, the penalty cost will be 0, α_0, α_1, α_2, α_3 respectively (see Fig. 4.1 and formula 4.4). That is to say, we have loose time windows. If c_1 and c_2 are bigger, patients give more flexibility to the decision-makers and caregivers. Three levels of ET are compared when the VD is $\delta * 2$ in Table 4.6 and 4.7. The results of the two methods do not dominate each other when the ET levels are small and medium. But the result of D*_MDLS is dominated by S_MDLS when $c_1 = 30$ and $c_2 = 45$. If patients have more flexibility, the S_MDLS is better to deal with uncertain service times.

TABLE 4.6 – Results of different levels of ET and VD when using S_MDLS.

	levels	D	NTC	NP	TC	P
ET						
Small	$l_i + 5, l_i + 15$	0.39	0.34	0.19	492.77	163.08
Medium	$l_i + 15, l_i + 30$	0.34	0.30	0.15	464.14	154.40
Large	$l_i + 30, l_i + 45$	0.37	0.32	0.20	**444.33**	**150.42**
VD						
Small	$\delta/3$	0.38	0.27	0.26	425.29	121.88
Medium	δ	0.50	0.39	0.31	446.94	143.72
Large	$\delta * 2$	0.37	0.32	0.20	**444.33**	**150.42**

TABLE 4.7 – Results of different levels of ET and V when using D*_MDLS.

	levels	D	NTC	NP	TC	P
ET						
Small	$l_i + 5, l_i + 15$	0.38	0.33	0.18	491.23	169.40
Medium	$l_i + 15, l_i + 30$	0.33	0.28	0.18	453.71	159.82
Large	$l_i + 30, l_i + 45$	0.40	0.37	0.16	**479.28**	**154.78**
VD						
Small	$\delta/3$	0.38	0.33	0.18	425.81	119.06
Medium	δ	0.57	0.37	0.43	436.65	156.08
Large	$\delta * 2$	0.40	0.37	0.16	**479.28**	**154.78**

Regarding VD, sensitivity analysis has been performed by increasing the variance of normal distribution. We sample the service times from normal distributions. If the variance is bigger which means patients are more likely to have larger or smaller service times that deviate from the average value, the results of S_MDLS dominate D*_MDLS (shown in the last rows of Table 4.6 and 4.7). Fig. 4.4 shows the Pareto points of D*_MDLS and S_MDLS with different VD and ET. The short lines in the box plots denote the median of TC or P. In (e), when $VD = \delta * 2$, $LD = (l_i + 30, l_i + 45)$, the median of S_MDLS is obviously smaller than D*_MDLS.

However, the S_MDLS is more realistic as it takes into account multiple scenarios, leading to increased computing time. If the variance of service times is not too large, the D_MDLS can be chosen to save computing time.

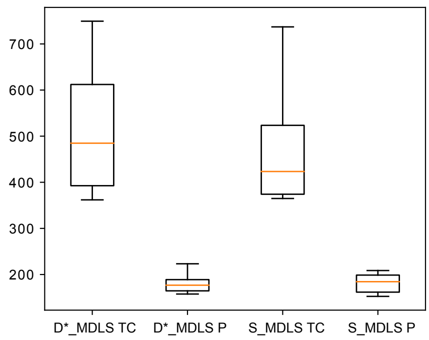
TABLE 4.8 – Objective values of trade-off solutions with changing of VD

Methods	$\delta/3$	δ	$\delta * 1.5$	$\delta * 2$	$\delta * 2.5$
Travel cost					
D*_MDLS	425.81	436.65	404.40	479.28	412.53
S_MDLS	425.29	446.94	414.51	444.33	413.67
Penalty					
D*_MDLS	119.06	156.08	161.34	154.78	170.64
S_MDLS	121.88	143.72	148.26	150.42	156.30

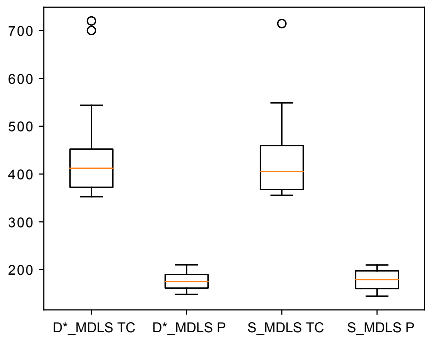
The values of travel cost and penalty of trade-off solutions for different approaches are shown in Table 4.8 and Fig. 4.5. The values of travel cost of the D*_MDLS are less stable than those of S_MDLS. When VD increases from $\delta/3$ to $\delta * 2.5$, the penalty of S_MDLS increases by 28.24%, while the growth for the D*_MDLS is 43.32%. The penalty of S_MDLS changes more sluggishly than that of D*_MDLS when VD changes. Therefore, S_MDLS has better robustness than the D*_MDLS. The objective values are more stable and perform better for the cases with large variations in service times by using the S_MDLS while D_MDLS is more appropriate for cases with small variations in service times to save computing time. Decision-makers can select one of the solutions from the Pareto sets depending on their companies' operating profitability. They can select which method to use in order to attain better objective values based on varying conditions.

4.4.2 Practical application for enhanced understanding

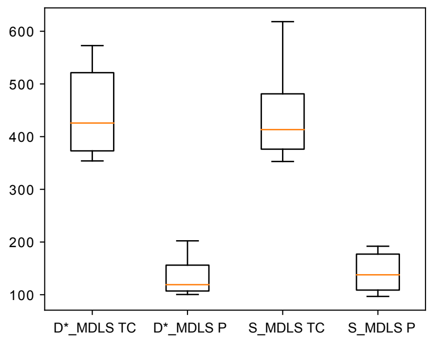
In this section, we apply the results derived from Section 4.4.1 on a real-life case and provide some actionable management recommendations. Using the real-life data provided by "Soins et Santé", a home health care company located in Lyon, France, we implement our methods to create routes and schedules. A total of 27 patients receive home care services. The available data set includes patients' locations,



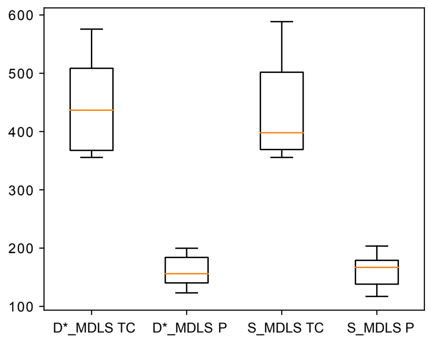
(A) $VD = \delta * 2, LD = (l_i + 5, l_i + 15)$.



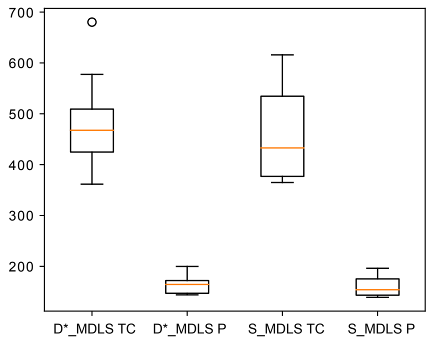
(B) $VD = \delta * 2, LD = (l_i + 15, l_i + 30)$.



(C) $VD = \delta / 3, LD = (l_i + 30, l_i + 45)$.

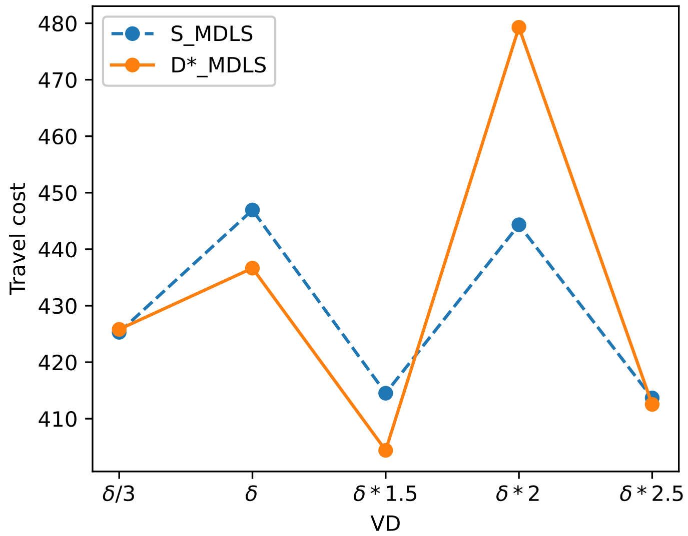


(D) $VD = \delta, LD = (l_i + 30, l_i + 45)$.

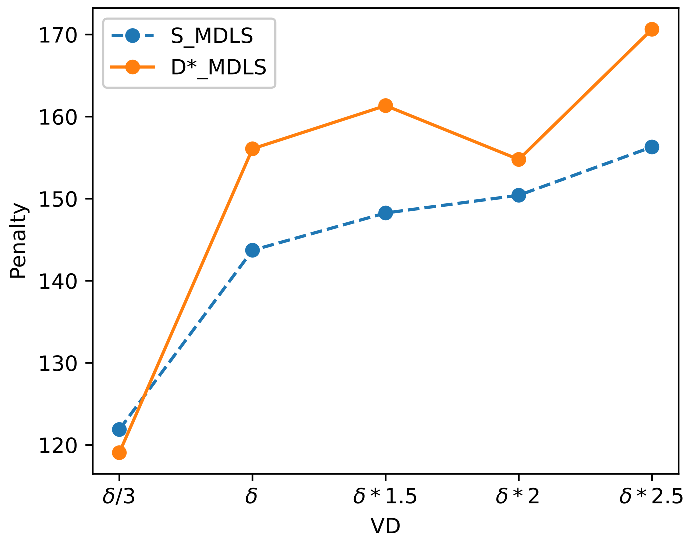


(E) $VD = \delta * 2, LD = (l_i + 30, l_i + 45)$.

FIGURE 4.4 – TC and P with different VD and ET.



(A) Travel cost.



(B) Penalty.

FIGURE 4.5 – Objective values of trade-off solutions with changing of VD.

service times, and available time sessions. The duration of each service varies, ranging from 5 to 46 minutes. The patients are visited in three time sessions: morning sessions from 7:30 to 12:00, afternoon sessions from 13:00 to 15:30, and evening sessions from 17:00 to 19:30. We create the time windows based on the preferred time sessions for visits. The length of the time windows ranges from 30 to 150 minutes. We also create the required level of caregiver for each patient.

Fig. 4.6 shows the results when $VD = \delta * 2$, $LD = (l_i + 30, l_i + 45)$, $m = 4$, and $n = 10$. In this figure, the Pareto front of the S_MDLS dominates that of the D*_MDLS algorithm. The S_MDLS is capable of generating a more diverse range of solutions. In real-life cases, when implementing the solution obtained by the D_MDLS, if the difference between the real service times and the planned service times is small, the actual objective function values are close to the original ones. However, if the real service times are significantly different from the planned service times, the solution obtained by the S_MDLS can be chosen to achieve smaller objective values on average. If it is not possible to determine the extent to which the patients' service times differ from the planned service times, the solution obtained by S_MDLS can be used by decision-makers to have higher stability.

By providing a more comprehensive view of the various solutions within the Pareto front, managers can gain a deeper understanding of the strengths and weaknesses of the solutions in the Pareto front. This knowledge can then be used to make decisions on selecting the most appropriate solution for their needs. We examine three points on the Pareto front, namely the solution with the minimum travel cost (S_1), the trade-off solution (S_2), and the solution with the minimum penalty (S_3). These points are clearly marked with three star icons within Fig. 4.6. We chose the trade-off solution with objective values closest to the origin. Table 4.9 presents the objective values and their respective indicators. The indicator PER_e represents the percentage of patients who are visited by caregivers before the earliest time of their time windows. The indicator PER_l denotes the percentage of patients whose services are completed by caregivers after the latest time of their time windows. The indicators WT_{min} and WT_{max} mean the minimum and maximum daily working hours, respectively. Although the minimum and maximum numbers of patients that each caregiver needs to visit (m and n) are limited in the proposed model, the solution S_3 achieves a better workload balance. If the majority of patients have a higher tolerance for exceeding their end time of the time windows, the manager may opt for a solution located on the left side of the Pareto front, which prioritizes minimizing travel costs. The routes corresponding to these solutions are visualized in Fig. 4.7. The locations of the patients (expressed by their longitude and latitude) and the planned routes are displayed on a map with a blank background to ensure their anonymity. The routes of S_3 are too complicated to be constructed manually, but they can achieve better satisfaction for patients and caregivers.

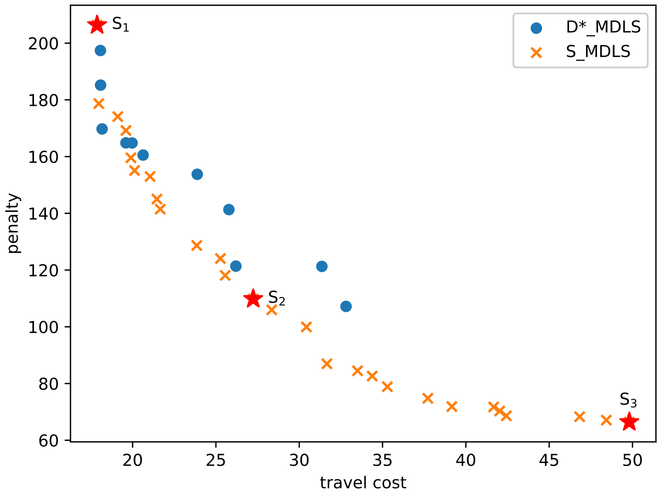
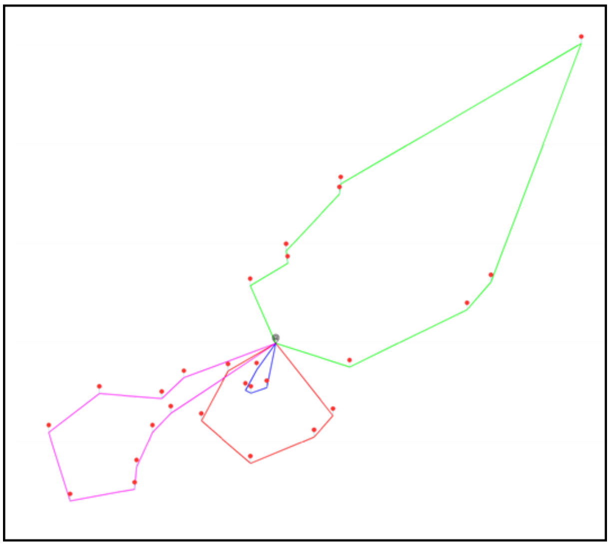


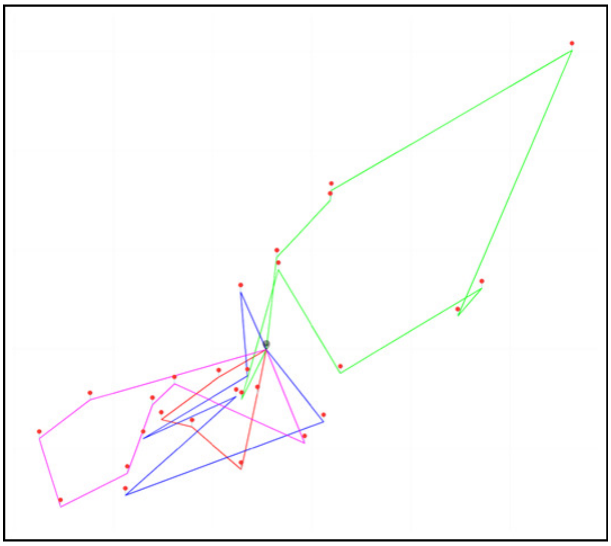
FIGURE 4.6 – Pareto front on a real-life case.

TABLE 4.9 – Indicators for three solutions

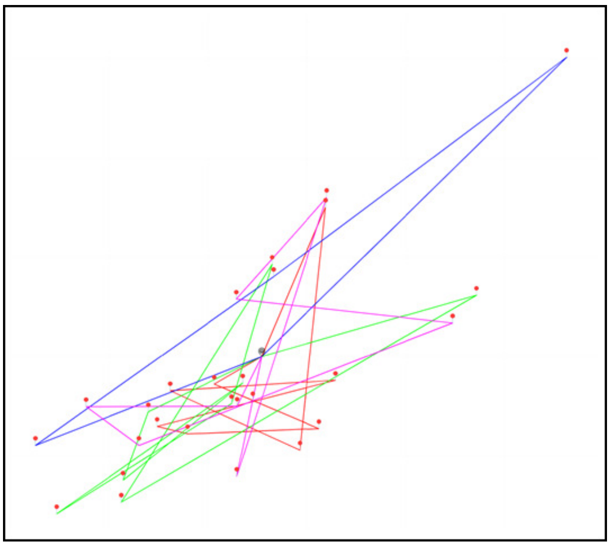
Solutions	f_1	f_2	PER_e	PER_l	WT_{min}	WT_{max}
S_1	17.87	206.36	28.96%	71.11%	298.99	806.27
S_2	27.24	109.82	52.30%	35.19%	424.29	714.94
S_3	49.81	66.42	45.33%	28.30%	263.57	677.99



(A) S_1



(B) S_2



(C) S_3

FIGURE 4.7 – Routes displayed on a blank background map.

4.5 Conclusions

This chapter addressed research questions Q3 and Q4. We developed a bi-objective model to optimize travel costs and satisfaction of both patients and caregivers, considering several practical constraints: the soft time windows, the matching of patient needs and caregiver skills, and the workload balance. We considered uncertain service times to enhance the practicality and robustness.

To solve the bi-objective optimization problem, we developed an ALNS-MDLS to obtain Pareto fronts. The Stochastic ALNS-MDLS (S_MDLS) was proposed to deal with the problem under the uncertain service times. First, we considered only one deterministic scenario: the average value of service times sampling from the normal distribution. The comparison between the Gurobi Solver and the ALNS-MDLS revealed the latter's superior efficiency and competitively high-quality solutions. Second, we considered uncertain service times, assuming they follow the normal distribution. In the D*_MDLS, the solutions obtained by the ALNS-MDLS were evaluated under uncertain service times. The results showed when the two parameters, i.e., ending time of the loose time and variance of distribution are bigger, the S_MDLS outperforms since its solutions dominate those of D*_MDLS. We evaluated the trade-off solutions of D*_MDLS and S_MDLS under varying variances. The outcomes confirmed S_MDLS's robustness, effectively demonstrating its efficacy in managing uncertain service times. Finally, a real-life application was conducted to provide practical managerial suggestions for choosing routes and schedules.

As a future development, we will create new heuristics and use exact methods to compare the results of this study. More practical objectives and constraints motivated by the needs of HHC companies can be added to further studies. We will try to accelerate the computing time, as the S_MDLS takes over twenty times longer to implement than the D_MDLS.

Chapter 5

Conclusions

This thesis addresses the weekly and daily routing and scheduling problems in home health care by proposing mathematical models and metaheuristic algorithms, using a stochastic approach to handle uncertain service times, and introducing multi-objective optimization problems to account for diverse and vague preferences. The effectiveness of these methods and models is proven through literature instances, real-world data, and computational experiments. The results indicate the benefits of considering uncertainty and differentiating between employees based on their contracts and customers' needs. This research has implications for home care managers and policymakers in the logistics industry, as it provides practical and innovative solutions for more efficient home care operations.

5.1 Key research questions and management suggestions

Four research questions were defined which are answered through Chapters 3 and 4.

— **Q1:** How the optimal weekly routes and schedules can be created while accommodating the needs of three different stakeholders?

In Chapter 3, a mixed-integer linear programming model was developed for weekly planning. The operational costs, penalty of dissatisfaction and workload difference are minimized. Different types of caregivers and patients with different levels of dependency are modeled. To solve the optimization problem efficiently, an ϵ-greedy neighborhood search heuristic algorithm is proposed. We generate instances based on operational data from a home care company in France, encompassing patient information such as geographic locations and levels of dependency, alongside the services they require on a weekly basis. The proposed method matches and exceeds the Pareto optimal solutions of the augmented ε-constraint (AUGMECON) method in small-sized instances and is also efficient in real-life scenarios. The results highlight that increasing operational costs in home health care can positively affect satisfaction and workload balance, emphasizing the need for careful cost management and stakeholder engagement for well-informed decision-making.

— **Q2:** What is the ideal number of each type of caregiver to hire, in order to effectively manage fluctuating task volumes?

Following the first question Q1, we categorize data into three groups, each with a different proportion of patients with high dependency, specifically 0%, 25%, and 50% respectively. We consider both external and internal caregivers with varying salaries and travel allowances. The focus is on optimizing the number of caregivers of each type. The workload increases with the rising number of high-dependency patients, necessitating the new assignment and scheduling of caregivers. The solutions obtained from the proposed method indicate that as operational costs are optimized, the number of internal caregivers either increases or remains in response to an increase in workload. With an increase in the number of high-dependency patients, more complex tasks can be allocated to internal caregivers. External caregivers can be assigned to the patients located nearby to manage the workload effectively and save operational costs.

— **Q3:** How can the robust daily routes and schedules be established while considering uncertain service times, as well as the satisfaction of both caregivers and patients?

In Chapter 4, we model the daily routing and scheduling problem using a mixed-integer linear formulation. The objectives are to minimize travel costs and penalties associated with caregiver and patient dissatisfaction. To solve this bi-objective optimization problem, we develop an Adaptive Large Neighborhood Search integrated with an enhanced Multi-Directional Local Search method (ALNS-MDLS). Additionally, uncertain service times are tackled by the stochastic ALNS-MDLS. Comparative analyses between the deterministic ALNS-MDLS and the stochastic ALNS-MDLS reveal that, despite the increased computational demands of the stochastic ALNS-MDLS, it excels in managing uncertain service times and demonstrates superior robustness when the real service times are significantly different from the planned service times.

— **Q4:** After we solve the problem and obtain solutions, how can we aid the managers in selecting a solution from the Pareto front in practice?

After addressing the third question Q3, our model and method are applied to a real-world case. We evaluate three solutions, namely the solution with the minimum travel cost (S_1), the trade-off solution (S_2), and the solution with the minimum penalty (S_3). The manager might choose a solution close to S_1 for cost reduction if most patients are flexible about appointment timings. The S_2 is ideal for balancing travel costs and time window penalties. The routes of S_3 are too complicated to be constructed manually, but our approach successfully facilitates this solution, thereby helping managers enhance the workload balance and the satisfaction of both caregivers and patients.

5.2 Perspectives

This thesis has several limitations. Firstly, it primarily focuses on balancing the needs of three key stakeholders in home care services, i.e., managers, caregivers and patients. However, family members and other informal caregivers also play a significant role sometimes in providing home care and in decision-making. For example, they collaborated with formal caregivers and concerned about hospital transfer decisions (Carretero et al., 2012; Pulst, Fassmer, and Schmiemann, 2019). Secondly, this thesis does not address the dynamic nature of patient needs changing over time, for example, the ending of contracts and the admission of new patients. Thirdly, the thesis predominantly relies on approximate algorithms for problem-solving, which means that optimal solutions for larger instances are not attainable. While the commercial solver Gurobi is effective in finding optimal solutions for small-sized instances, the time required to solve large-scale data is substantially greater. Finally, handling instances with more than 100 patients in Chapter 3 and solving the stochastic model in Chapter 4 are notably time-intensive, indicating the need for enhanced computational efficiency.

Future research directions stemming from this thesis could delve deeper into several aspects. In terms of modeling, there is a need to broaden the scope to include a wider range of stakeholders, exploring their impact on the home care setting, along with their needs, preferences, and interactions. The work satisfaction of caregivers can be addressed, as overtime for nurses often leads to reduced effectiveness and can negatively impact patient outcomes. Nurse-to-patient ratios, as discussed in Davis et al. (2014), and the duration of continuous work shifts for nurses could be optimized. The weekly planning would be extended to long-term planning, focusing on dynamic resource reallocation for efficiently integrating newly admitted patients. The focus for long-term planning could be to maintain the original schedule to the greatest extent possible. A volatile schedule will affect voluntary turnover, while a stable schedule can attract caregivers and ensure the continuity of care for patients (Bergman, David, and Song, 2023). On the other hand, there is considerable scope for enhancing algorithm performance, both in terms of solution quality and computational speed. Refining these algorithms by incorporating methods such as column generation and machine learning is recommended. Furthermore, comparing more techniques to manage uncertainties would advance the overall effectiveness of the proposed solutions.

Appendix A

Algorithms

Algorithm 9-12 show the detail of the worst destroy operator, the relatedness destroy operator, the greedy repair operator and the regret repair operator.

Algorithm 9 Worst destroy operator

Input: a solution x, the number of nodes to be removed q

Output: the removal list D

while $q > 0$ **do**
  L contains all nodes of the solution
  for each node i in x **do**
    remove node i from x
    $cost_i \leftarrow f(x) - f(x_{-i})$
  end for
  sort L in descending order of $cost_i$
  random number y_q in interval [0,1)
  $d \leftarrow L[y_q|L|]$
  remove d from x
  $D \leftarrow D \cup d$
  $q = q - 1$
end while

Algorithm 10 Relatedness destroy operator

Input: a solution x

Output: the removal list D

$d \leftarrow random\_choose(x)$
while $q > 0$ **do**
  $d \leftarrow$ random_choose(R)
  for each node i in x **do**
    $relatedness_i \leftarrow cal\_relatedness(i, d, x)$
  end for
  sort L in descending order of $cost_i$
  random number y_q in interval [0,1)
  $d \leftarrow L[y_q|L|]$
  remove d from x
  $D \leftarrow D \cup d$
  $q = q - 1$
end while

Algorithm 11 Greedy repair operator

Input: the solution x' without the nodes in the removal list D

Output: a new solution x^{new}

while $|D| > 0$ **do**
  for each node d in D **do**
    for each insert position $i \leftarrow 1$ to $|route| - 1$ **do**
      insert node d at position i
      $cost(d, i) \leftarrow f(x') - f(x'_{+i})$
    end for
  end for
  find smallest $cost(d, i)$ and its corresponding d and i
  $x^{\text{new}} \leftarrow insert(d, i, x')$
  remove node d from D
end while

Algorithm 12 Regret repair

Input: the solution x' without the nodes in the removal list D, $regret\_n$

Output: a new solution x^{new}

```
while |D| > 0 do
    for node d in D do
        L ← ∅
        for each insert position i ← 1 to |route| − 1 do
            insert node d at position i
            cost(d, i) ← f(x'_{+i})
            L ← L ∪ cost(d, i)
        end for
        g ← 0
        L ← sort_ascending(L)
        for i ← 1 to regret_n do
            g ← g + L[i] − L[0]
        end for
        find the biggest g and its corresponding d and i
    end for
    x^new ← insert(d, i, x')
    remove node d from D
end while
```

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FOLIO ADMINISTRATIF

THESE DE L'INSA LYON, MEMBRE DE L'UNIVERSITE DE LYON

NOM : ZHAO DATE de SOUTENANCE : 13/05/2024
(avec précision du nom de jeune fille, le cas échéant)

Prénoms : Jiao

TITRE : Optimisation Multi-Objectifs à Court et Moyen Terme dans la Planification des Soins de Santé à Domicile

NATURE : Doctorat Numéro d'ordre : 2024ISAL0038

Ecole doctorale : Ecole Doctoral Informatique et Mathématiques (N° ED512)

Spécialité : Informatique et applications

RESUME :
L'industrie des Soins de Santé à Domicile (SSD) offre des soins essentiels aux personnes âgées, handicapées et malades chroniques, financée par l'assurance sociale et la fiscalité. Les entreprises SSD doivent planifier efficacement pour maximiser l'utilisation des ressources et assurer des soins de qualité.
Dans les entreprises de SSD, les gestionnaires acceptent un nombre limité de patients, évaluant leur niveau de dépendance et planifiant leurs services hebdomadaires. Les soignants, internes et externes, visitent les patients selon des itinéraires et horaires définis. L'objectif est de créer ces itinéraires et horaires tout en considérant le nombre de soignants différents. Une approche de programmation linéaire en nombres entiers mixtes est utilisée, intégrant une recherche de grand voisinage dans un cadre de recherche locale améliorée. Les résultats montrent une performance supérieure à la méthode augmentée de contrainte et enfin des recommandations de gestion sont données.
Suite à la planification hebdomadaire en soins à domicile, des incertitudes liées aux temps de service peuvent survenir, affectant la qualité du service. Pour y remédier, nous introduisons un problème d'optimisation bi-objectif pour la planification et le routage incertains. Nous proposons des versions déterministes et stochastiques d'une recherche adaptative de grand voisinage intégrée dans un cadre de recherche locale multidirectionnelle améliorée, offrant une efficacité supérieure comparée au Solveur Gurobi. La robustesse de notre modèle et méthode est confirmée par une analyse de sensibilité. Enfin, l'application pratique de cette méthode est démontrée par un cas réel, accompagnée de recommandations managériales.

MOTS-CLÉS : Optimisation multi-objectifs, Recherche de grand voisinage, Problèmes de routage et de planification des véhicules, incertitude, Soins de santé à domicile.

Laboratoire (s) de recherche : Laboratoire DISP EA4570

Directeur de thèse: MONTEIRO Thibaud

Président de jury :

Composition du jury :

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DI MASCOLO, Maria	Directrice de Recherche Grenoble INP	Examinatrice
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